

MATH 324A (Autumn 2009)
Practice Midterm

Student name: _____

Student number: _____

Signature: _____

Do not start working until instructed to do so.

You have 50 minutes.

Please show your work.

Scientific, but not graphing calculators are allowed.

You may use one 8.5 by 11 double-sided sheet of handwritten notes.

Problem 1 (20 points)	
Problem 2 (10 points)	
Problem 3 (10 points)	
Problem 4 (10 points)	
Total	

Problem 1 (20 points) Evaluate the following integrals.

(a) $I = \int_D e^{x^2+y^2} dA$, where $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2\}$.

(b) $I = \int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1} dy dx$.

(c) $I = \int_E x^2 + y^2 + z^2 dV$, where $E = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1, z \geq 0\}$.

(d) $I = \int_D x^2 dA$, where $D = \{(x, y) \in \mathbb{R}^2 : (x - 2)^2 + (y - 3)^2 \leq 1\}$.

Problem 2 (10 points) Find the volume of the solid

$$E = \left\{ (x, y, z) \in \mathbb{R}^3 : (x^2 + y^2)^{\frac{1}{4}} \leq z \leq 1 \right\}.$$

Problem 3 (10 points) Find the center of mass of a lamina which has constant density $\rho(x, y) \equiv 1$ and occupies the region $D = [0, 1] \times [0, 1] \cup \{(x, y) \in \mathbb{R}^2 : (x - \frac{1}{2})^2 + (y - 1)^2 \leq \frac{1}{4}\}$. (The region is the union of a unit square and a semi-disk.)

Problem 4 (10 points) Consider the change of variables

$$\begin{cases} x = u + 2v \\ y = u - 2v \end{cases}$$

(a) Verify that the image of the unit circle $u^2 + v^2 = 1$ under the above transformation is the ellipse $5x^2 + 6xy + 5y^2 - 16 = 0$. (You don't need to explain why it is an ellipse.)

(b) Evaluate the double integral $I = \int_R \sqrt{4(x+y)^2 + (x-y)^2} dA$, where R is the region bounded by the ellipse in the previous part.