MATH 324 C
Exam II
November 19, 2012

Name ______________________________

Student ID #____________________ Section ____________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

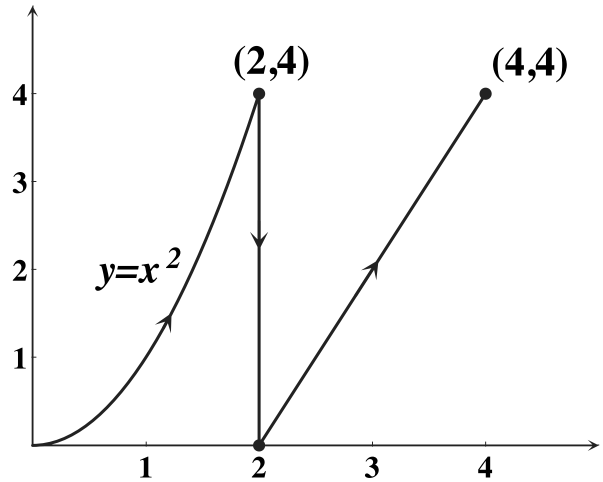
SIGNATURE:______________________________

1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- Your exam should consist of this cover sheet, followed by 5 problems. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 5 problems. Try not to spend more than 10 minutes on each page.
- Show all your work and justify your answers.
- Your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use a scientific calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing calculators) are forbidden.
- The use of headphones or earbuds during the exam is not permitted.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

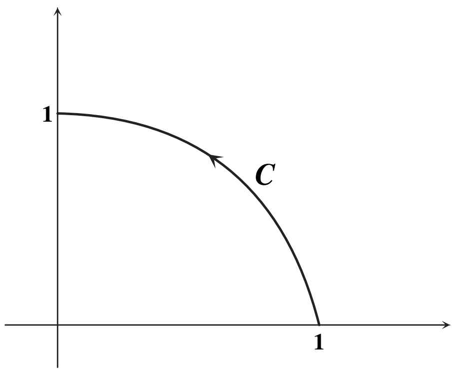
GOOD LUCK!

1. (10 points) A wire is bent in the shape pictured below: a portion of the parabola $y = x^2$ followed by two line segments. At the point (x, y) the density of the wire is $\rho(x, y) = x$. Find the mass of the wire.



2. (10 points)

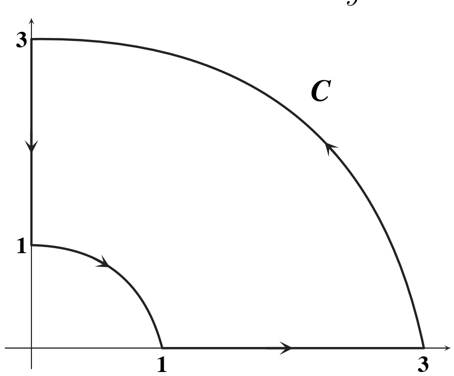
(a) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \left\langle x^2 + \frac{1}{y}, e^y \right\rangle$ and C is the quarter-circle $x^2 + y^2 = 1$ in the first quadrant traversed counter-clockwise.



(b) Evaluate the line integral

$$\oint_C (e^x + 6xy)\, dx + (8x^2 + \sin y^2)\, dy,$$

where C is the curve consisting of the arcs of the quarter-circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ in the first quadrant, along with the line segments that join the arcs along the x- and y-axes as shown below.



3. (10 points) Let $\mathbf{F}(x, y, z) = \langle x^3 - 3xy^2, y^3 - 3x^2y, 30z \rangle$.

   (a) Find a function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.

   (b) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the portion of the helix

$$x = 3\cos t, y = 3\sin t, z = t, \text{ for } 0 \le t \le \pi.$$

4. (10 points) The temperature in degrees Celsius at the point (x, y) is given by the function

$$T(x, y) = \sqrt{2x^2 - y}.$$

(a) A heat-seeking bug sits on the xy-plane at the point $(3, 2)$. In what direction should the bug travel to increase its temperature at the fastest rate? (Express your answer as a vector in $\mathbb{R}^2$.)

(b) What is that rate? (Assume x and y are measured in centimeters and include units in your answer.)

5. (10 points)

(a) Let $\mathbf{F}(x, y, z) = (\cos x - z)\,\mathbf{i} + y^2\,\mathbf{j} + xz\,\mathbf{k}$.

i. Find a point (x, y, z) at which $\operatorname{curl}\mathbf{F} = \mathbf{j}$.

ii. Is there a vector field $\mathbf{G}$ such that $\mathbf{F} = \operatorname{curl}\mathbf{G}$? Explain.

(b) Give a parameterization $x = x(u, v),\ y = y(u, v),\ z = z(u, v)$ of the plane that contains the points $(0, 1, 1)$, $(1, 2, 3)$, and $(2, 2, 1)$.

(c) Let $f(x, y, z) = x^3 + yz^2$ and suppose $x = x(s, t)$, $y = y(s, t)$, and $z = z(s, t)$. The table below gives the values of x, y, and z at the point $(s, t) = (-1, -1)$ as well as their partial derivatives at that point. Compute $\dfrac{\partial f}{\partial s}$ at the point $(s, t) = (-1, -1)$.

	at the point $(s, t) = (-1, -1)$
x	1
y	0
z	1
x_s	-2
x_t	1
y_s	1
y_t	-2
z_s	-1
z_t	-1