Math 324 - Winter 2012
Exam 2
February 24, 2012

Name: ______________________________

Student ID Number: ______________________________

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- There are 6 questions spanning 5 pages. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 10 MINUTES PER PAGE!

GOOD LUCK!

1. (10 pts) Consider the vector field $\mathbf{F}(x, y, z) = x^2y\,\mathbf{i} + zy\,\mathbf{j} + yx^3\,\mathbf{k}$ on $\mathbb{R}^3$.

   (a) (6 pts) Compute the following:

      i. $\operatorname{curl}\mathbf{F}$.

      ii. $\nabla(\operatorname{div}\mathbf{F})$.

      iii. $\operatorname{div}(\operatorname{curl}\mathbf{F})$.

   (b) (4 pts) Give a short one sentence answer to each of the two questions below:

      i. What can you conclude for a vector field where $\operatorname{curl}\mathbf{F} \neq \mathbf{0}$?

      ii. What can you conclude for a vector field where $\operatorname{div}\mathbf{F} \neq 0$?

2. (8 pts)

(a) (4 pts) Give a parameterization for the part of surface $y^2 + z^2 - x = 3$ with $0 \leq x \leq 1$. Include bounds on the parameters.

(b) (4 pts) You are told that $x = x(t)$, $y = y(t)$, and $z = z(t)$ is the parameterization for the motion of some particle along the curve C which is on the surface $z = x^2 + \sin(y) - xy^2$. If $x(1) = 2$, $y(1) = 0$, $x'(1) = 3$, and $y'(1) = -5$, then what is the value of $z'(1)$?
That is, find $\frac{dz}{dt}$ at $t = 1$.

3. (9 pts) Consider the vector field $\mathbf{F}(x, y, z) = (-z\sin(x) + y^2)\mathbf{i} + (2xy + e^{z^2})\mathbf{j} + (\cos(x) + 2yze^{z^2})\mathbf{k}$ on $\mathbf{R}^3$. You are told that the vector field is conservative!

(a) (6 pts) Find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$.

(b) (3 pts) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ over the curve, C, given by $\mathbf{r}(t) = \langle \pi t, 3 - 3t^4, \sin(\pi t) + 5t \rangle$ for $0 \leq t \leq 1$. (Please think about your options here.)

4. (8 pts) Use Green's Theorem to evaluate

$$\oint_C \sin(x^3)\,dx + 4x^2y\,dy$$

where C is the triangle with vertices $(0,0)$, $(2,0)$, and $(2,6)$.

5. (5 pts) Assume the temperature at each point on the xy-plane is given by

$$T(x,y) = \frac{1}{3}x^2y + 5\sqrt{x^2+y^2} \quad \text{degrees Celcius,}$$

where x and y are in feet. Find the directional derivative of $T(x,y)$ at the point $(3,4)$ in the direction of $\langle -1, 2\rangle$. **Give the units for your answer**.

6. (10 pts) Assume, again, the temperature at each point on the xy-plane is given by $T(x,y) = \frac{1}{3}x^2y + 5\sqrt{x^2+y^2}$ degrees Celcius. You are told that the average temperature along a curve C is given by $\frac{1}{L}\int_C T(x,y)\,ds$, where L is the total length of C.

Let C be the curve consisting of a straight line segment from the origin to $(0,2)$, then one quarter of the circle $x^2+y^2=4$ from $(0,2)$ to $(2,0)$. Compute the average temperature along C. That is, compute $\frac{1}{L}\int_C T(x,y)\,ds$.

(Hint: Parameterize!)