

Math 324 - Winter 2012

Exam 1

January 27, 2012

Name: _____

Student ID Number: _____

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- There are 5 questions spanning 5 pages. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 10 MINUTES PER PAGE!

GOOD LUCK!

1. (8 pts) Reverse the order of integration and evaluate

$$\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1} dx dy.$$

2. (12 pts) Consider the solid region between $z = x$ and $z = x^2$. Let E be the solid that is within this region and bounded between the planes $y = 0$ and $y + 6z = 6$.

(a) Set up the triple integral $\iiint_E 1 \, dV$ in each of the specified orders

i. $dydzdx$:

ii. $dx dz dy$:

(b) Find the volume of E .

3. (10 points) Let E be the solid bounded in the **first octant** by $x^2 + y^2 = 9$ and $z = y$. Assume the density of the solid is a constant $\rho(x, y, z) = 6 \text{ kg/m}^3$. Use cylindrical coordinates to find the z -coordinate of the center of mass. (Hint: I'll tell you that the volume of E is 9 m^3).

4. (9 points) Let E be the part of the solid bounded between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ with $z \leq 0$ and $y \geq 0$.

(In other words, below the xy -plane and on the positive y side of the xz -plane).

Use spherical coordinates to evaluate $\iiint_E \frac{1}{\sqrt{x^2 + y^2}} dV$

5. (11 points) Note: Parts (b) and (c) below are unrelated to Part (a).

(a) (3 pts) Compute the Jacobian, $\frac{\partial(x,y)}{\partial(u,v)}$ for the transformation $x = 3u^2 + v^2$ and $y = uv^2$.

(b) (3 pts) Find the inverse of the transformation: $x = 2u + 2v$ and $y = -2u + 2v$.

(c) (5 pts) Consider the triangular region, R , in the xy -plane bounded by $(0, 0)$, $(4, 0)$ and $(4, 4)$.
A picture of this region is below.

Sketch a detailed graph in the uv -plane of the image of R under the transformation:

$$x = 2u + 2v \text{ and } y = -2u + 2v.$$

(Label the new corners and sides).

