Math 324 - Fall 2013
Exam 2
November 15, 2013

Name: _____

Student ID Number: _____

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- There are 4 questions spanning 4 pages. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 10 MINUTES PER PAGE!

GOOD LUCK!

1. (13 pts) Felix is walking on the surface $z = f(x, y) = \frac{1}{4}x^2 + \frac{1}{3}y^3 - \frac{1}{2}xy^2$, where x, y and z are in miles. Label the positive y-direction as NORTH and the positive x-direction as EAST.

   (a) Felix's x and y coordinates are given by $x = x(t)$ and $y = y(t)$ where t is in hours since he started walking. We are told that $x(1) = 2$, $y(1) = 1$, $x'(1) = 3$ and $y'(1) = -4$.
   At $t = 1$ hour, find the rate of change of Felix's height with respect to time.
   (Give units for your answer)

   (b) Later Felix stops and takes a break at the point $(x, y) = (8, 2)$. Give the **unit** direction vector that points in the (x, y) direction Felix needs to initially walk in order to go steepest **downhill**.

   (c) Felix is still standing at $(x, y) = (8, 2)$. He decides to walk in the direction that is 30 degrees east of north. Find the slope in this direction.

2. (12 pts) Consider the vector field $\mathbf{F}(x, y) = \langle xy, -x^2y \rangle$ on $\mathbb{R}^2$.
Note: For any computations that require a z-component, assume the z-component is zero.

(a) Compute $\operatorname{curl} \mathbf{F}$.

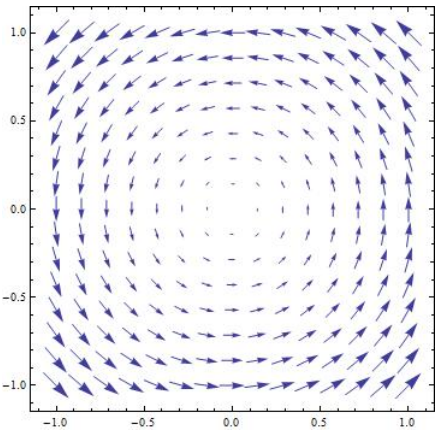
(b) Compute $\operatorname{div} \mathbf{F}$

(c) Give an example (any example) of a point (x_0, y_0) in the vector field at which the $\operatorname{curl} \mathbf{F}$ points in the negative z direction (into the page). And tell me if the vector field has a tendency to rotate clockwise or counterclockwise about the point you have given.

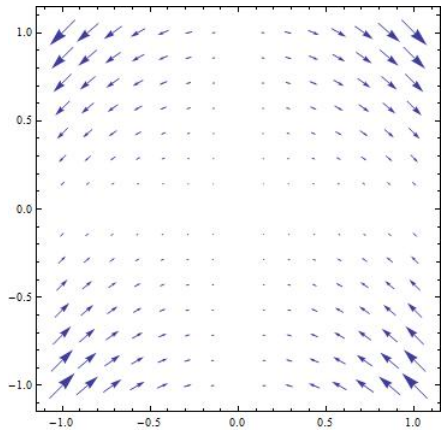
Circle one: Clockwise or Counterclockwise.

(d) Circle the picture that corresponds to this vector field.

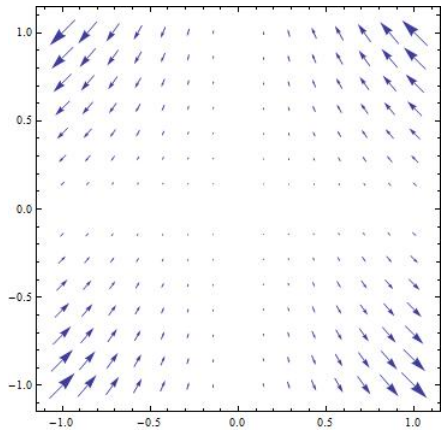
I



II



III



3. (a) (7 pts) Let C be the closed loop given by first following $y = 3x^2$ from $(0, 0)$ to $(1, 3)$, then following the straight line back from $(1, 3)$ to $(0, 0)$.

   Using any appropriate method, evaluate $\oint_C (2xy^2 + \sin(x))\,dx + (y - y^3)\,dy$.

   (b) (6 pts) The base of a fence is the circle of radius 2 in the first quadrant. The height of the fence at position (x, y) is given by the function $h(x, y) = 3 + x$. Give the area of one side of the fence.

4. (12 pts) Consider the vector field $\mathbf{F} = \langle -y^2 \sin(xy^2), -2xy \sin(xy^2) + 2e^{2y}, \sqrt[3]{z} \rangle$.
This vector field is conservative!

(a) Find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$.

(b) Let C_1 be the line from $(0, 0, 0)$ to $(1, 2, 8)$.
Let C_2 be the curve parameterized by $x = 1+t-t^2$, $y = 2-2t^4$, $z = 8+\sin(\pi t)$ for $0 \leq t \leq 1$.
Let C be the curve given by C_1 followed by C_2.
Using any appropriate method, evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$. (Please use your fastest option!).