Print your name: ______________________________

This exam has 6 questions on 5 pages, worth a total of 50 points.

Problem	Points	Score
1	4	
2	6	
3	10	
4	10	
5	10	
6	10	
Total	50	

You should:

- write complete solutions or you may not receive credit.
- $\boxed{\text{box your final answer.}}$

You may:

- use one sheet of notes and a non-graphing calculator.
- write on the backs of the pages if you need more room.

Please do not:

- come to the front of the room to ask questions (raise your hand).
- share notes or calculators.
- use any electronic device other than a calculator.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ______________________

1. (4 points) Suppose $f(x, y) = x^2y + y^2$, and $x = x(u, v)$ and $y = y(u, v)$ are functions of u and v, with

$$x(1,2) = 3 \qquad \frac{\partial x}{\partial u}(1,2) = -1 \qquad y(1,2) = 1 \qquad \frac{\partial y}{\partial u}(1,2) = 2,$$

Find $\dfrac{\partial f}{\partial u}$ when $u = 1$ and $v = 2$.

2. Let $g(x, y) = x \sin y$.

(a) (3 points) Determine the directional derivative $D_{\mathbf{u}}g(1, 0)$ if $\mathbf{u} = \left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.

(b) (3 points) Find a unit vector $\mathbf{v}$ so that $D_{\mathbf{v}}g(1, 0) < -\frac{3}{4}$.

3. (10 points) Let E be the solid bounded by the following four planes:

$$x = 0 \qquad y = 0 \qquad z = 0 \qquad 2x + 2y + z = 4$$

Find the x-coordinate of the center of mass if the solid has constant density.

4. Consider the region whose volume is naturally given by the integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} 1\,dz\,dy\,dx.$$

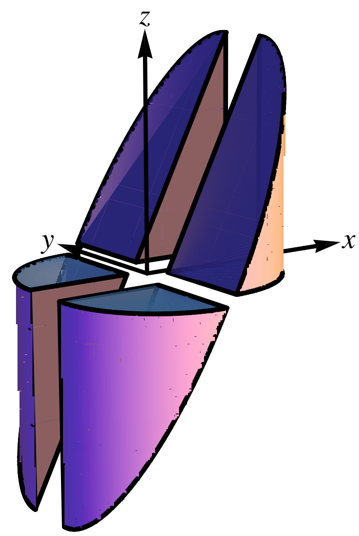
   (a) (5 points) Write an integral in cylindrical coordinates that computes the volume of the same region. *Do not evaluate the integral.*

   (b) (5 points) Write an integral in spherical coordinates that computes the volume of the same region. *Do not evaluate the integral.*

5. (a) (8 points) Compute the integral

$$\iiint_R yz^2\,dV$$

where R is one of the four (you choose) regions bounded by the cylinder $x^2+y^2=1$ and the three planes $z=2x$, $z=0$, and $y=0$, as shown below.



(b) (2 points) Now use symmetry to determine the value of the same integral over each of the 4 regions:

	$x \le 0, z \le 0$	$x \ge 0, z \ge 0$
$y \le 0$		
$y \ge 0$		

6. (10 points) Compute the integral

$$\int_0^1 \int_0^{1-x} \exp\left(\frac{x-y}{x+y}\right) dy\, dx$$

using the change of coordinates $u = x - y$, $v = x + y$.
Note that exp is the exponential function: $\exp a = e^a$.