Math 324B
SECOND PRACTICE EXAM

1. Suppose $w = f(u, v)$, $u = \dfrac{\tan z}{x^2} + 2\ln y$, and $v = (2x - 3y)^3$.
   a. Compute $\partial w/\partial x$, $\partial w/\partial y$, and $\partial w/\partial z$ in terms of the partial derivatives of f.
   b. Considering w as a function of x, y, and z, what is its maximum directional derivative at $(x, y, z) = (2, 1, \frac{1}{4}\pi)$, given that $\nabla f(\frac{1}{4}, 1) = 4\mathbf{i} - \frac{1}{3}\mathbf{j}$? (Note that $(u, v) = (\frac{1}{4}, 1)$ when $(x, y, z) = (2, 1, \frac{1}{4}\pi)$.)

2. Let $f(x, y, z) = x^2 y/z + \cos \pi xy + e^{2x-3z}$.
   a. Compute $\nabla f(x, y, z)$.
   b. Find a unit vector $\mathbf{u}$ of the form $\mathbf{u} = a\mathbf{j} + b\mathbf{k}$ such that $D_{\mathbf{u}} f(3, 1, 2) = 0$.

3. Compute $\int_C (y/x)\, ds$ where C is the line segment from $(1, 5)$ to $(3, 7)$.

4. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = y\mathbf{i} + 2x\mathbf{j}$ and C is the portion of the curve $x = \cos y$ from $(1, 0)$ to $(1, 2\pi)$.

5. Let $\mathbf{F}(x, y, z) = e^{-2y}\mathbf{i} + (z^3 - 2xe^{-2y})\mathbf{j} + 3(y + 1)z^2\mathbf{k}$.
   a. Find a function f such that $\mathbf{F} = \nabla f$.
   b. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is parametrized by $\mathbf{r}(t) = (1 - t^4)\mathbf{i} + (\sin \pi t^2)\mathbf{j} + \sqrt{3t^3 + 1}\,\mathbf{k}$, $0 \le t \le 1$. (Don't do it the hard way!)

6. Let S be the piece of the surface $z = 2xy$ where $x^2 + y^2 \le 4$.
   a. Find the area of S.
   b. Let $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + e^{(x+y)^2 - z}\mathbf{k}$. Evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is given the *downward* orientation. (Hint: The exponential term simplfies.)