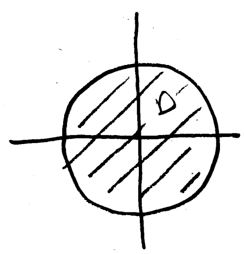
# Clarification

A **simple region** is a region in the xy-plane that we can integrate with respect to x **or** y first without the integral breaking into multiple pieces... (abstractly). (see Ex 2)

What exactly does that mean? Let's look at a few examples:

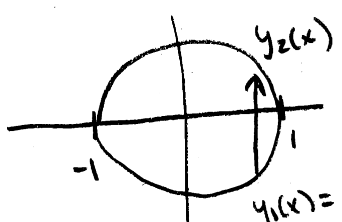
Ex 1 Region $\{(x,y) \mid x^2+y^2 \le 1\}$.




simple region !

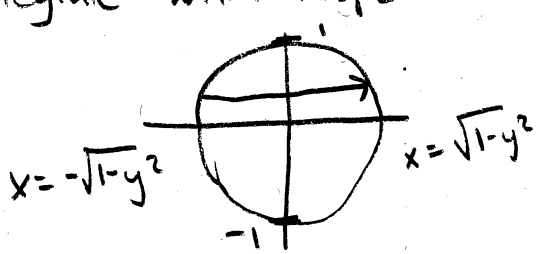
why?

Integrate a function over the region with respect to y first:



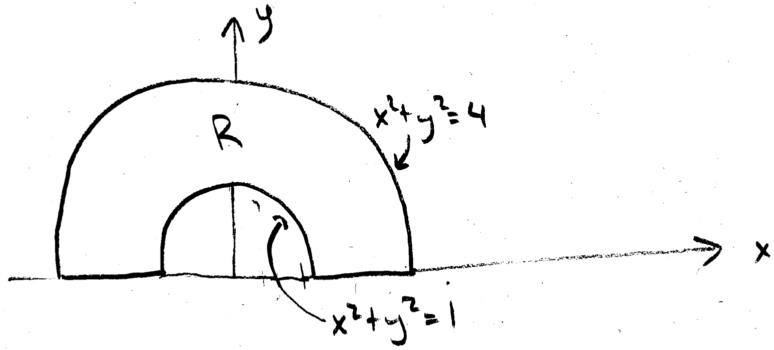

$$\int_{x=-1}^{x=1} \int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} f(x,y)\, dy\, dx$$

Integrate with respect to x first:

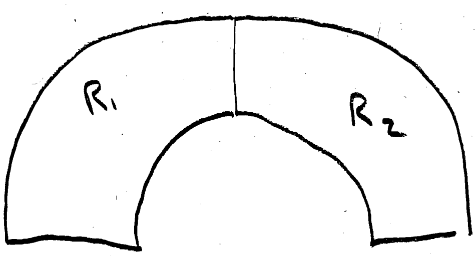



$$\int_{y=-1}^{y=1} \int_{x=-\sqrt{1-y^2}}^{x=\sqrt{1-y^2}} f(x,y)\, dx\, dy$$

## Ex 2 (Not Simple)

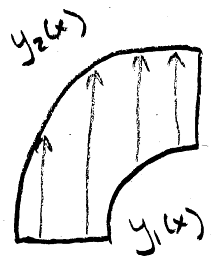



Cannot integrate with respect to $x$ first without the region breaking up ... **BUT** with one division, it becomes a "simple region".




$R = R_1 + R_2$

Take $R_1$, integrate with respect to $y$ first:




$y_2(x) = \sqrt{4-x^2}$

$$y_1(x) = \begin{cases} 0, & -2 < x < -1 \\ \sqrt{1-x^2}, & -1 < x < 0 \end{cases}$$

Then $\displaystyle\int_{x=-2}^{x=-1}\int_{y_1(x)}^{y_2(x)} f(x,y)\,dy\,dx$ is good enough.

Do you object because $y_1(x)$ is only piecewise smooth? You should, but ... →

even though when you compute this integral, it will break into two integrals, when you handle the integral abstractly by applying the Fundamental Theorem of Calculus to it, we only need for $y_1(x)$ and $y_2(x)$ to be continuous. This is the subtlety, and really just a techinicality.

Why do we define simple regions this way?

Short answer, when proving Green's Theorem, you have to apply the fundamental theorem of calculus twice across a region, once with respect to the x-variable, and a second time with respect to the y-variable. So the proof works for simple regions. But, as the theorem was stated in class, we don't restrict ourselves to simple regions. Why? Our regions can be subdivided into simple regions, and Green's Theorem applied to each piece, which, in effect, is just Green's Theorem applied to non-simple regions.