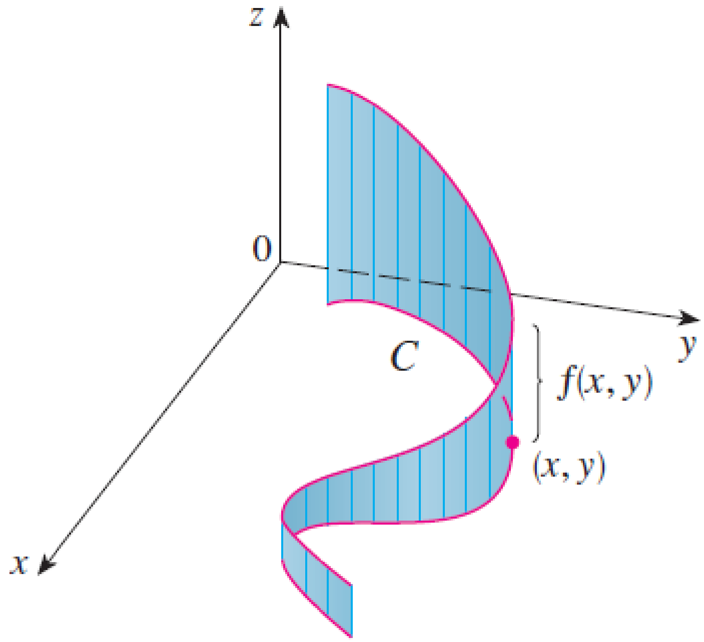
Visualization of a <u>Line Integral</u> of a <u>Scalar Field</u> in R^2



$$\int_C f(x, y)\, ds = \int_a^b f\big(x(t), y(t)\big)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

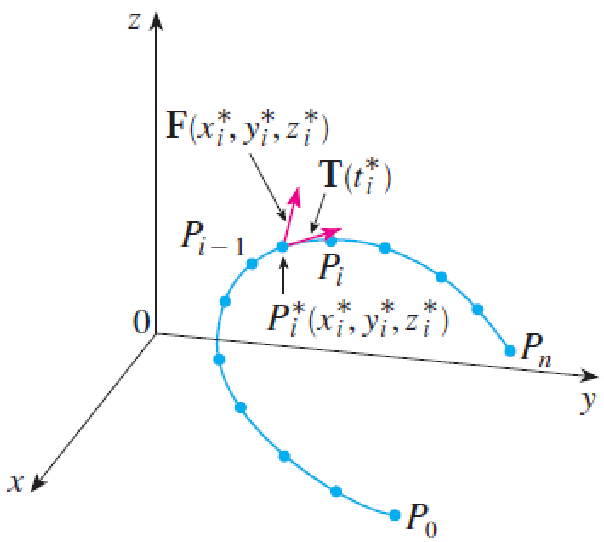
Applications:

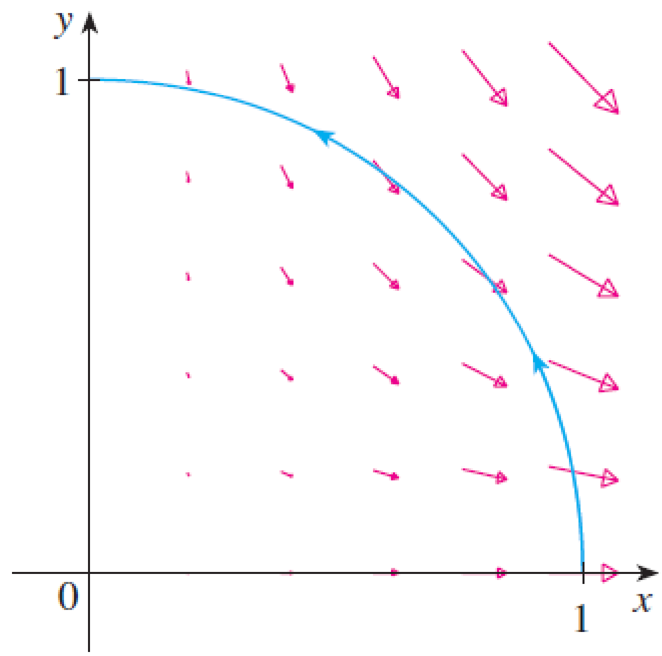
Area of one side of the shape shown.

Average value.

Center of Mass.

Visualizations of a Line Integral of a Vector Field





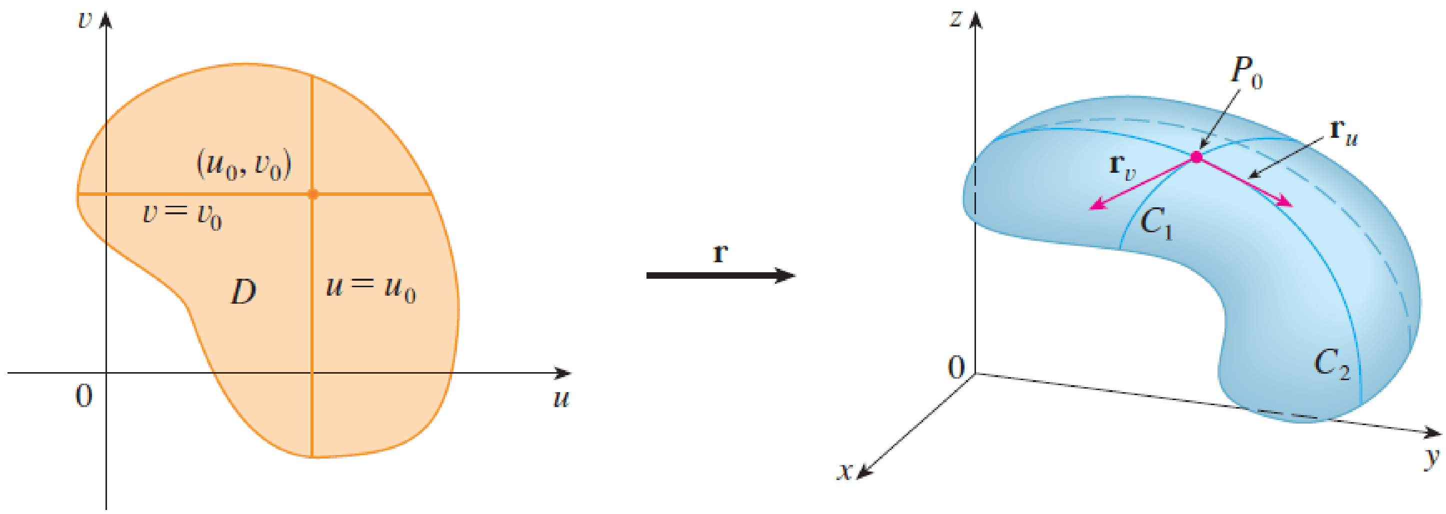
$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt = \int_C \mathbf{F} \cdot \mathbf{T}\, ds$$

Applications:

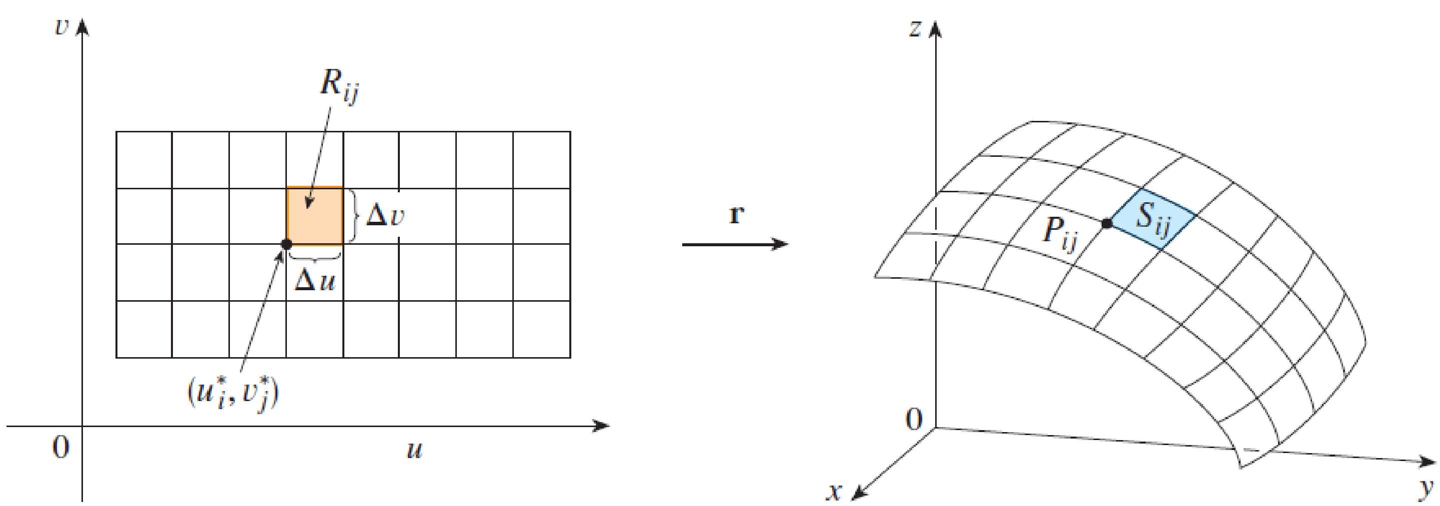
Work done by the vector field on an object moving along the curve.

The flow of the vector field in the direction of the curve.

Visualizations of a Surface Parameterization Facts

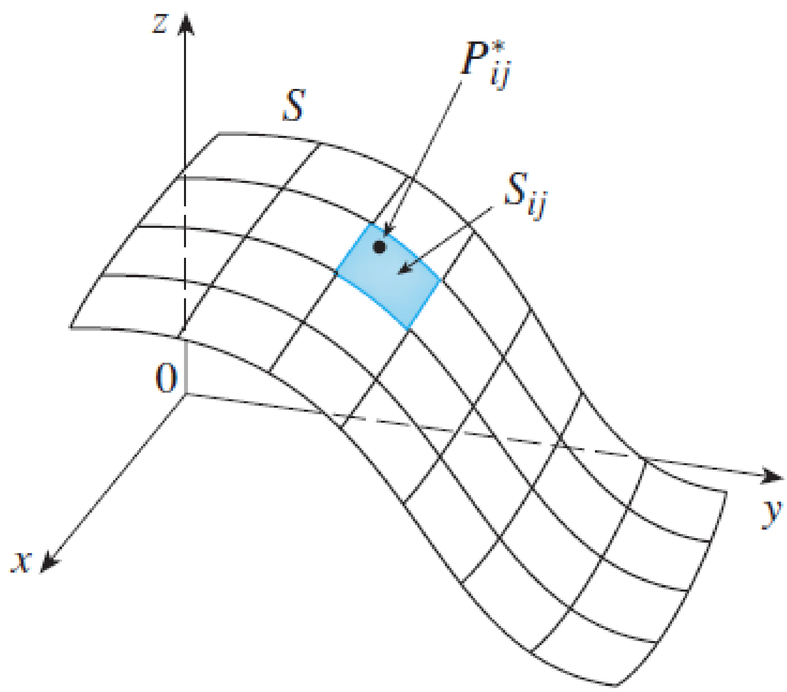


$$\mathbf{r}(u, v) = x(u, v)\,\mathbf{i} + y(u, v)\,\mathbf{j} + z(u, v)\,\mathbf{k}$$



$$A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| \, dA$$

Visualization of a Surface Integral of a Scalar Field in R^3



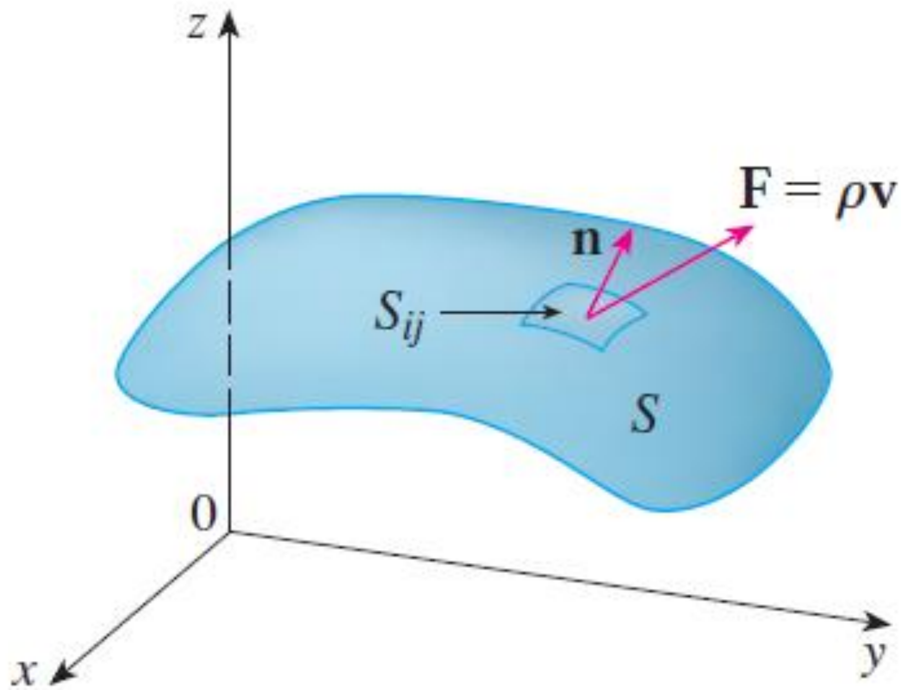
$$\iint_S f(x, y, z)\, dS = \lim_{m, n \to \infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(P_{ij}^*)\, \Delta S_{ij}$$

$$\iint_S f(x, y, z)\, dS = \iint_D f(\mathbf{r}(u, v))\, |\mathbf{r}_u \times \mathbf{r}_v|\, dA$$

Applications:

Surface area, Average value, center of mass.

Visualizations of a **Surface Integral** of a **Vector Field**



$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS \qquad \mathbf{n} = \frac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}$$

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dA$$

Applications and Interpretations of a Surface Integral of a Vector Field

The surface integral of a vector field measure the "flux" across the surface.

- For a velocity vector field of a fluid, this gives the rate of flow through the surface.
- For an electric field, this gives the electric flux through the surface. Guass' Law states that net charge on a closed surface is

$$Q = \varepsilon_0 \iint_S \mathbf{E} \cdot d\mathbf{S}$$

- If u(x,y,z) is the temperature at each point, then the heat flow is defined to be the vector field $\mathbf{F} = -K\nabla u$ and the rate of heat flow across the surface S is given by

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = -K \iint_S \nabla u \cdot d\mathbf{S}$$