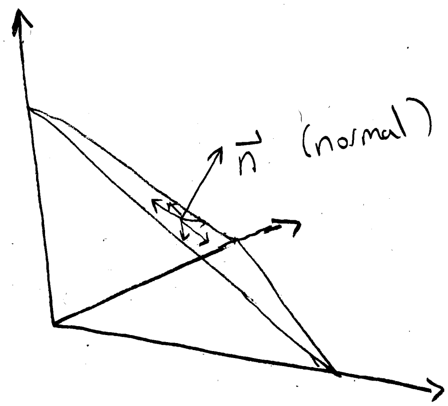
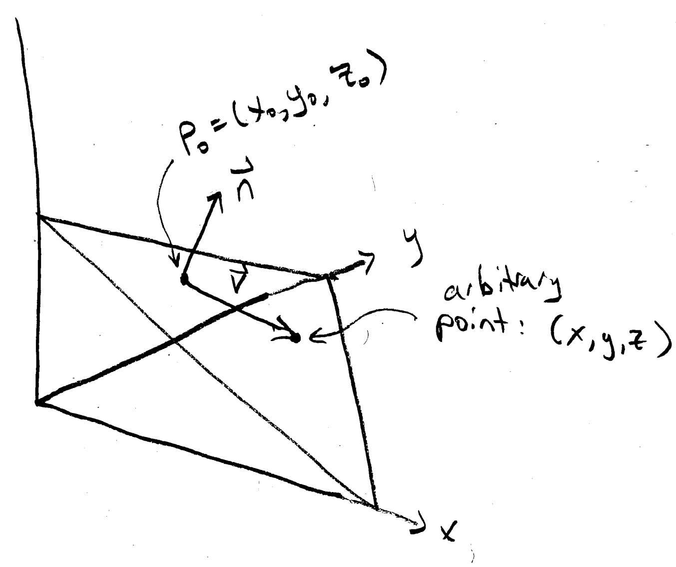
Equation of a Plane

Given a plane, if you have a normal vector, then ~~any~~ any vector in the plane (parallel to the plane) is perpindicular to the normal vector:



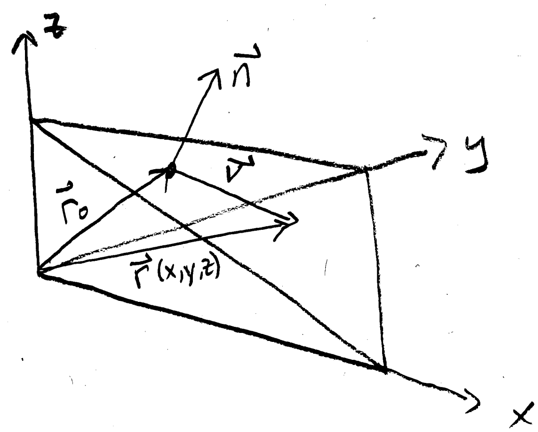
We can use this fact to write the scalar equation of a plane. Assume you have a point $P_0 = (x_0, y_0, z_0)$ in the plane! and a normal vector. Let $\vec{v}$ be a vector starting



at P_0 pointing at any arbitrary point in the plane. Then $\vec{v}$ is parallel to the plane, and we can write:

$$(*) \quad \vec{n} \cdot \vec{v} = 0.$$

Can we describe vector $\vec{v}$ a little better? Yes!



Let $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$, and notice $\vec{r}(x,y,z) = \langle x, y, z \rangle$ points to our arbitrary point. Then, we can write

$$\vec{v} = \vec{r} - \vec{r}_0$$

so our equation $(*)$ becomes

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

Writing this out, let $\vec{n} = \langle a, b, c \rangle$ and notice

$$\langle a,b,c \rangle \cdot (\langle x,y,z \rangle - \langle x_0, y_0, z_0 \rangle) = 0$$

$$\langle a,b,c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$ax + by + cz - ax_0 - by_0 - cz_0 = 0$$

$$ax + by + cz = \underbrace{ax_0 + by_0 + cz_0}_{\text{call this } d}$$

$$(*) \quad \boxed{ax + by + cz = d} \quad (*)$$

So, given a normal vector $\vec{n} = \langle 1, 2, 3 \rangle$, we know the scalar equation of the plane has the form

$$1 \cdot x + 2 \cdot y + 3 \cdot z = d.$$

If you have a point on the plane, plug it in for (x, y, z) to compute d. For example, if the point $(2, 1, 0)$ was on the plane with $\vec{n} = \langle 1, 2, 3 \rangle$, then

$$1(2) + 2(1) + 3(0) = d$$

$$2 + 2 = d$$

$$4 = d$$

and the equation of the plane is

$$\boxed{x + 2y + 3z = 4}.$$