

Math 308G/H - Winter 2018
Midterm 2
2018-02-21

Name: _____
Student ID Number: _____

Question	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
Total:	60	

- There are 5 problems on this exam. Be sure you have all 5 problems on your exam.
- The final answer must be left in exact form. Box your final answer.
- You are allowed the TI-30XIIS calculator. It is possible to complete the exam without a calculator.
- You are allowed a single sheet of 2-sided handwritten self-written notes.
- You must show your work to receive full credit. A correct answer with no supporting work will receive a zero.
- Use the backsides if you need extra space. Make a note of this if you do.
- Do not cheat. This exam should represent your own work. If you are caught cheating, I will report you to the Community Standards and Student Conduct office.

Conventions:

- I will often denote the zero vector by 0 .
- When I define a variable, it is defined for that whole question. The A defined in Question 2 is the same for each part.
- I treat row and column vectors as the same.
- For any linear transformation T , there exists a matrix A such that $T(x) = Ax$. I defined the determinant, rank, and nullity of T using A . This means,

$$\det(T) = \det(A), \quad \text{rank}(T) = \text{rank}(A), \quad \text{nullity}(T) = \text{nullity}(A).$$

1. Give an example of each of the following. If it is not possible, write "NOT POSSIBLE". You do not need to justify your answers.

(a) (3 points) If possible, give an example of a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ with $\text{rank}(T) = 3$.

(b) (3 points) If possible, give an example of a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ with $\text{nullity}(T) = 3$.

(c) (3 points) If possible, give an example of a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that T is one-to-one but T^2 is not onto.

(d) (3 points) If possible, give an example of a basis for \mathbb{R}^3 where each basis element lies in the $2x + y + z = 1$ plane. (Hint: Note that $1 \neq 0$.)

2. (a) (6 points) Suppose $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation with the property that

$$T(2, 0) = (3, 0), \quad T(1, 1) = (0, 2).$$

What is $|\det(T)|$? (Hint: A picture could be helpful here.)

- (b) (6 points) Let

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 21 \\ 3 & 8 & 2018 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & -3 \end{bmatrix}, C = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- What is $\det(B)$?

- What is $\det(C)$?

- What is $\det(2A^{-1}B^TAC^2)$? Here B^T denotes the transpose of B . You may leave your answer as a product of numbers and their powers.

3. Let A and C be equivalent matrices given by

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \\ 3 & 0 & 6 \\ 4 & 8 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = C.$$

Let T be the linear transformation defined by $T(x) = Ax$.

(a) (6 points) For each of the following subspaces, write down a basis for the subspace. If you write your answer as a matrix, I will draw frowny faces on your exam and will consider taking off points

- $\text{col}(A)$

- $\text{ker}(T)$

- $\text{row}(A)$

(b) (6 points) Let $U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. You do not need to justify the following answers.

- What is $\text{rank}(T)$?
- What is $\text{nullity}(AU)$?
- What is $\text{nullity}(2C^T)$?

4. Let A and C be equivalent matrices defined by

$$A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & 1 & 3 & 3 \\ 3 & 1 & 4 & 0 \\ 4 & 1 & 5 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = C.$$

(a) (4 points) What is a basis for $\text{row}(A)$?

(b) (4 points) Denote the basis for $\text{row}(A)$ found in the previous part by \mathcal{B} . If possible, write the following vectors as linear combinations of the element of \mathcal{B} . Otherwise, write “NOT POSSIBLE” and justify why is it not possible.

- $(2, 2, 4, 0)$

- $(2, 1, 0, 1)$

(c) (4 points) Hopefully, you determined that \mathcal{B} does not span \mathbb{R}^4 . Give an example of a vector that could be added to \mathcal{B} so that together they span \mathbb{R}^4 . Be sure to justify your answer.

5. Let

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, u_2 = \begin{bmatrix} 3 \\ 0 \\ 8 \end{bmatrix}, u_3 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}.$$

Let $U = [u_1 \ u_2 \ u_3]$ and $\mathcal{B} = \{u_1, u_2, u_3\}$. We have that U is invertible. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation with the property:

$$T(u_1) = u_1, \quad T(u_2) = 2u_2, \quad T(u_3) = 2u_1.$$

From lecture, we know that

$$T(x) = U \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^{-1}x.$$

(a) (4 points) You do not need to show work for the following quick questions:

- What is $\text{rank}(T)$?

- What is $\det(T)$?

- What is $\text{nullity}(T)$?

- Is T invertible?

(b) (4 points) What is $T(2u_1 + u_2)$? You may express your answer as a linear combination of u_1, u_2, u_3 .

(c) (4 points) Suppose $[x]_{\mathcal{B}} = (2, 1, 0)$. What is $[T(x)]_{\mathcal{B}}$? (Hint: a intermediate step could be to determine what x is. Then look at the previous part.)