Your Name

Student ID #

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- Check that you have a complete exam. There are 5 questions for a total of 50 points.
- You are allowed to have one handwritten note sheet. Only basic non-graphing scientific calculators are allowed, though you should not need one.
- **Cheating will result in a zero and be reported to the Dean's Academic Conduct Committee.**
- **Show all your work**. Unless explicitly stated otherwise in a particular question, if there is no work supporting your answer, you will not receive credit for the problem. If you need more space to answer a question, continue on the back of the page, and indicate that you have done so.

Question	Points	Score
1	8	
2	16	
3	8	
4	8	
5	10	
Total:	50	

1. Multiple choice questions. You are **not required to show any work**.

   (a) (1 point) If A and B are invertible $n \times n$ matrices, then $(A+B)^{-1} = A^{-1} + B^{-1}$.
   ◯ True ◯ False

   (b) (1 point) How many 2×2 matrices A satisfy $A^2 = I_2$?
   ◯ 1 ◯ 2 ◯ 4 ◯ Infinitely many

   (c) (1 point) Every subspace is the null space of some matrix.
   ◯ True ◯ False

   (d) (1 point) A subspace $S \neq \{\mathbf{0}\}$ can have a finite number of vectors.
   ◯ True ◯ False

   (e) (1 point) If W is a subspace of $\mathbb{R}^n$ such that $\dim(W) = n$, then W has to be the whole space $\mathbb{R}^n$.
   ◯ True ◯ False

   (f) (1 point) If A is the matrix of transformation corresponding to the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that rotates $\mathbb{R}^2$ by $30°$ (i.e. 30 degrees) about the origin, then which of the following is true? (I_2 denotes the usual 2×2 identity matrix.)
   ◯ $A^3 = I_2$ ◯ $A^6 = I_2$ ◯ $A^{12} = I_2$ ◯ $A^9 = I_2$

   (g) (1 point) What is the nullity of the matrix $A = \begin{bmatrix} 5 & 17 & 2017 \end{bmatrix}$?
   ◯ 0 ◯ 1 ◯ 2 ◯ 3

   (h) (1 point) Let A, B be $n \times n$ matrices, let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, and let s, t be scalars. Which of the following are *always true*? (Check all that apply.)
   ◯ $(AB)^2 = A^2B^2$. ◯ $(A+B)^2 = A^2 + 2AB + B^2$.
   ◯ $A\mathbf{0} = \mathbf{0}$. ◯ $A^2 = A$ implies $A(A - I_n) = 0$, so either $A = I_n$ or $A = \mathbf{0}_{nn}$.

2. Short answer questions. Please **justify** any work briefly.

   (a) (3 points) Give an example of two subspaces S_1 and S_2 of $\mathbb{R}^4$ each of dimension 2 where the only vector belonging to both S_1 and S_2 is $\mathbf{0}$.

   (b) (4 points) Verify if the subset S in $\mathbb{R}^4$ consisting of vectors of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ satisfying $x_1 + 2x_2 + 3x_3 = 0$ is a subspace.

(c) (4 points) Let $A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Compute A^4.

(d) (5 points) Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that the range of T is the plane $x + y - z = 0$. Does T have to be one-to-one?

3. Let A be the following 3×3 matrix:

$$A = \begin{bmatrix} 1 & 4 & -3 \\ 1 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) (5 points) Compute A^{-1}.

(b) (3 points) Show that A^T is invertible, with $(A^T)^{-1} = (A^{-1})^T$.

4. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that the following hold.

$$T\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix}$$

(a) (5 points) Find $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right)$ for arbitrary real numbers a and b.

(b) (3 points) Find the matrix of transformation A corresponding to T, i.e., find the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^2$.

5. Let A be the following 3×5 matrix. Its reduced echelon form B is provided.

$$A = \begin{bmatrix} 1 & 1 & 7 & 0 & 0 \\ 3 & 1 & 15 & 6 & 0 \\ 0 & 2 & 6 & 3 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & 3 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = B.$$

(a) (4 points) Let T be the linear transformation having A as the matrix of transformation. Write down its domain and codomain. Is T one-to-one? Onto?

(b) (4 points) Find a basis for null(A), and compute the nullity of A.

(c) (2 points) Find another basis for null(A) different from the one you obtained in (b) above.