Your Name

Your Signature

Student ID #

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	

Question	Points	Score
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	

1. (10 points) Let A be the matrix $\begin{bmatrix} 2 & 1 & 3 & 1 & 0 \\ 0 & 2 & -1 & 1 & 1 \\ 4 & 0 & 8 & 1 & 1 \\ -2 & 1 & -3 & 0 & 3 \end{bmatrix}$.

   Calculate the rank and the nullity of A.

2. (10 points) Define a linear transformation T by the formula

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+2y+3z \\ y+z \\ -3x+7y+4z \end{bmatrix}$$

   Determine if T is invertible. If it is, give a formula for T^{-1}.

3. (10 points) For each part, find 2×2 matrices A and B satisfying the conditions indicated.
   (a) $A^2 = B^2$, but A is not a scalar multiple of B.
   (b) $AB = 0$ but $BA \neq 0$.

4. (10 points) Let A be a 4×4 matrix. Let V be the set of vectors $\mathbf{v} \in \mathbb{R}^4$ with the property that $A\mathbf{v} = A^2\mathbf{v}$. Determine whether V is a subspace of $\mathbb{R}^4$. Justify your answer.

5. (10 points) Find vectors $\mathbf{u_1}$, $\mathbf{u_2}$ and $\mathbf{u_3}$ such that $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly independent but $\{3\mathbf{u_1}, 5\mathbf{u_2} - \mathbf{u_1}, 2\mathbf{u_3} + \mathbf{u_2} - 7\mathbf{u_1}\}$ is linearly dependent, or explain why such vectors cannot exist.

6. (10 points) Let W be the null space of the matrix $\begin{bmatrix} 1 & 0 & -1 & 2 \\ -2 & 1 & 3 & -7 \\ 0 & -1 & -1 & 3 \\ 1 & 0 & -1 & 2 \end{bmatrix}$.

   Calculate a basis for $W^\perp$.

7. (10 points) Define a linear transformation T by the formula

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+3y-5z \\ x+y+9z \\ z \\ 3y+z \end{bmatrix}$$

Calculate an orthogonal basis for the range of T.

8. (10 points) Find all values of x for which the matrix

$$A = \begin{bmatrix} x & -2 & 1 \\ x & 4 & -2 \\ 2+x & 6 & 3x \end{bmatrix}$$

is *not* invertible.

9. (10 points) Let $A = \begin{bmatrix} 2 & -18 & -9 \\ 0 & -4 & -3 \\ 0 & 6 & 5 \end{bmatrix}$.

   Find the eigenvalues of A. Then find bases for the corresponding eigenspaces of the matrix.

10. (10 points) Find a 2×3 matrix A whose null space is $\text{span}\{(2, 1, -1)\}$. (Hint: Find a basis for the orthogonal complement of $\text{span}\{(2, 1, -1)\}$.)

11. (10 points) Let T be the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ where A is the matrix

$$A = \begin{bmatrix} -2 & -1 & 1 & -4 \\ 0 & 1 & 3 & 0 \\ 1 & 0 & -2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Is T onto? Carefully justify your answer.

12. (10 points) Find a vector in $\mathbb{R}^3$ that is not in the span of $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$. Carefully verify your solution.