Your Name

Your Signature

Student ID #

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	

Question	Points	Score
5	10	
6	10	
7	10	
8	10	
Total	80	

1. (10 points) Find a vector in $\mathbb{R}^3$ that is not in the span of $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$. Carefully verify your solution.

2. (10 points) Find the least squares solution(s) for the given linear system.

$$\begin{aligned} x_1 - x_2 &= -3 \\ x_1 + 2x_2 &= 2 \\ x_1 \phantom{{}+2x_2} &= 1 \end{aligned}$$

3. (10 points) Define a linear transformation T by the formula

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3y-2z \\ y-z \\ x+2y+5z \end{bmatrix}$$

Determine if T is invertible. If it is, give a formula for T^{-1}.

4. (10 points) Let T be the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ where A is the matrix

$$A = \begin{bmatrix} -2 & -1 & 1 & -4 \\ 0 & 1 & 3 & 0 \\ 1 & 0 & -2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Is T onto? Carefully justify your answer.

5. (10 points) Let V be the set of vectors of the form $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, where $a+2b=0$ and $3c-d=0$.

   Determine whether V is a subspace of $\mathbb{R}^4$. If it is a subspace, give a basis for it. Justify your answer.

6. (10 points) Find vectors $\mathbf{u}$, $\mathbf{v}$ such that $\{\mathbf{u},\mathbf{v}\}$ is linearly independent but $\{\mathbf{u}-\mathbf{v}, 2\mathbf{u}+\mathbf{v}\}$ is linearly dependent, or explain why such vectors cannot exist.

7. (10 points) Let $A = \begin{bmatrix} 2 & -18 & -9 \\ 0 & -4 & -3 \\ 0 & 6 & 5 \end{bmatrix}$.

   Find the eigenvalues of A. Then find bases for the corresponding eigenspaces of the matrix.

8. (10 points) Let $\mathbf{u_1} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{u_2} = \begin{bmatrix} 3 \\ 1 \\ -3 \end{bmatrix}$, $\mathbf{v_1} = \begin{bmatrix} -4 \\ -1 \\ 5 \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} 13 \\ 5 \\ -11 \end{bmatrix}$,

   Determine whether $\operatorname{span}\{\mathbf{u_1}, \mathbf{u_2}\} = \operatorname{span}\{\mathbf{v_1}, \mathbf{v_2}\}$. Carefully justify your answer.