NAME ____________________

Math 308E
Spring 2016

Midterm 1
April 20, 2016

Instructions

- Point totals for each problem are shown in parentheses.
- You must show all your work on the examination to receive credit. You must also use the techniques of this course on each problem; if you have taken a course in linear algebra previously, you may not use anything from that course which was not covered here. Ask if you are unsure of what is allowed.
- Read each problem carefully. You will not receive credit if you misunderstand or misread a problem.
- Your work must be neat and organized.
- Be very careful with your arithmetic. None of the calculations or answers are too complicated.
- Make sure your test has 5 questions.

(6) 1. Find all solutions to the system of linear equations

$$\begin{aligned} 6x_1 + 4x_2 + 3x_3 - 3x_4 &= 12 \\ 2x_1 + x_2 - x_4 &= 4 \\ -2x_1 + 4x_3 - x_4 &= -1 \end{aligned}$$

(6) 2. Find all values c for which the equation

$$\begin{bmatrix} 2 & 4 & -1 \\ 1 & 5 & 1 \\ 0 & 2 & 1 \\ 1 & 4 & c \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

has no solutions, exactly one solution, and an infinite number of solutions.

(6) 3. Let $\mathbf{v}_1 = (1, 2, 1)$, $\mathbf{v}_2 = (3, -1, 1)$, and $\mathbf{v}_3 = (-1, 5, 1)$ be vectors in $\mathbf{R}^3$. Find a vector not in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. (You must show how you know that the vector you find is not in the span; just writing down a vector is not sufficient.)

(4) 4. Find a 3×4 matrix A, in *reduced* echelon form, with free variable x_3, such that the general solution of the equation $A\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$ is

$$\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 6 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix},$$

where s is any real number.

(3) 5. Find vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ in $\mathbf{R}^3$ such that $\{\mathbf{u}_1, \mathbf{u}_2\}$, $\{\mathbf{u}_1, \mathbf{u}_3\}$, and $\{\mathbf{u}_2, \mathbf{u}_3\}$ are all linearly independent but $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ isn't, or explain why such vectors cannot exist.