Final Exam Concept Checklist

Important note: The final exam will be cumulative. I have eliminated chapter 1 from this list. There will not be any "solve the following linear system problems" but I do of course expect you to be able to apply the basic procedures of solving linear systems to, for example, find a basis for a null space.

Chapter 2

- I understand addition and scalar multiplication of vectors algebraically and geometrically. I know the algebraic properties that vector addition and scalar multiplication satisfy. I know how to multiply an nxm matrix A by a vector x in $\mathbb{R}^m$.
- I can state the definition of the span of a set of vectors. I can determine if a vector in $\mathbb{R}^n$ is in the span of a set of vectors. I can determine if a set of vectors spans $\mathbb{R}^n$.
- I can state the definition of linear dependence and linear independence.
- I can determine whether a set of vectors is linearly independent or linearly dependent.
- I understand the connection between solving a linear system and determining if a vector is in the span of other vectors.
- I have a conceptual understanding of linear dependence and linear independence. I can rephrase the definition of linear independence or dependence in terms of span and in terms of solutions to linear equations.
- I understand this statement: The following are equivalent.

Chapter 3

- I can determine whether a function $T : \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation. I understand that a linear transformation is completely determined by its values on $e_1, e_2, \ldots, e_m$ and that every linear transformation can be represented by a matrix.
- Given a linear transformation T, I can find the matrix A associated to T.
- I can graphically represent linear transformations in $\mathbb{R}^m$. I know the matrices for linear transformations from $\mathbb{R}^2$ to itself, dilating, rotating, and reflecting the unit square. (This is all in the Lecture 3.1 notes towards the end).
- Given a linear transformation T, I can determine whether it is one-to-one and whether it is onto. I understand how a linear transformation T being one-to-one or onto translates into properties of the matrix associated to T.
- I can perform algebraic operations with matrices, including addition, subtraction, scalar multiplication, and matrix multiplication. I can compute the transpose of matrices, and know how to identify diagonal and triangular matrices. I understand the connection between matrix multiplication and composition of linear transformations.
- I can find the inverse of a matrix or determine that no inverse exists using Gaussian elimination.
- I understand what it means for a matrix to be invertible, both algebraically and conceptually. I understand the relationships among a matrix being invertible, properties of the associated linear transformation, and spanning or linear independence properties of the columns of A (e.g., the Unifying Theorem v. 3).
- I understand the subtleties of what arithmetic properties break down with matrices (i.e. $AB \neq BA$), and how these can be resolved if a matrix is invertible (for example, see Graham's Winter 2019 Q4 in Final Exam Archive 1).

Chapter 4

- I can show whether or not a subset W forms a subspace. I can determine and characterize subspaces of $\mathbb{R}^n$. I have a geometric understanding of what subspaces look like in $\mathbb{R}^2$ and $\mathbb{R}^3$.
- I can rigorously show whether a subset of vectors from a subspace forms a spanning set for the subspace (or not). I can show whether a subset of vectors from a subspace is linearly independent (or not). I can determine whether a set of vectors forms a basis for a subspace, and if not, I can find a basis for it (see Method 1 and Method 2 in the 4.2 Lecture notes). I can find the dimension of a subspace. I can identify a basis for $\mathbb{R}^2$ and $\mathbb{R}^3$ pictorally.
- I can find a basis for the row space, the column space, or the null space of a matrix. I can determine the rank and nullity of a matrix and apply the rank-nullity theorem in appropriate situations.
- I can find a basis for the kernel and range of a linear transformation. I can rephrase statements about a linear transformation being one-to-one or onto in terms of statements about the kernel and range.
- I can find a change of basis matrix to express a vector in one basis, in terms of another. (This includes moving to/from non-standard bases from/to standard bases, as well as moving from non-standard bases to non-standard bases.)
- I can express a linear transformation of $\mathbb{R}^2$ (given in the standard basis) in terms of another basis.
- If given a coordinate vector in a non-standard basis, I can express the vector in the standard basis.

Chapter 5

- I can compute the determinant of 2×2 and 3×3 matrices using the definition or the visual method.
- I understand how elementary row operations change the determinant for $n \times n$ matrices. I can compute the determinant of an $n \times n$ matrix using row reduction.
- I know how to use the determinant to test whether a matrix is invertible.
- I know how the determinant of a product of matrices relates to the determinant of each factor, how the determinant of A and A^{-1} are related, and shortcuts for computing the determinant of upper (or lower) triangular matrices.
- I can compute the determinant of an $n \times n$ matrix using cofactor expansion.
- I know how the determinant relates to area/volume in the context of linear transformations, and I can apply this mode of thinking to appropriate problems.

Chapter 6

- I can compute the characteristic polynomial of any matrix. Given a factored characteristic polynomial of a matrix, I can compute the eigenvalues and eigenvectors of a matrix. I know the relationship between the dimension of an eigenspace and the multiplicity of an eigenvector.
- I can compute the eigenvalues and eigenspaces of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ and with $n = 2$ or 3 and I can describe them geometrically.
- Given a matrix A and an eigenvalue λ of A, I can compute the eigenspace, E_λ, of λ. I can show that the eigenspace is a subspace.
- I understand how eigenvalues and associated eigenvectors related to rotations and reflections in $\mathbb{R}^n$ and can find planes of reflection/axes of rotation if given the appropriate problem.

- Given an $n \times n$ matrix A with with n linearly independent eigenvectors, I can describe the linear transformation associated to A for $n = 2$ or 3.
- I understand why an $n \times n$ matrix A matrix is diagonalizable if and only if A has eigenvectors that form a basis for $\mathbb{R}^n$.
- Given a $n \times n$ matrix A that has eigenvectors that form a basis for $\mathbb{R}^n$, I can diagonalize A.
- I understand the relationship between finding eigenvalues and computing determinants of matrices of the form $A - \lambda I_n$.
- I understand how the process of diagonalization relates to expressing a linear transformation in a basis of eigenvectors.
- I understand reflections and rotations in the setting of eigenvalues and eigenspaces.