MATH 308 O
Exam II
May 15, 2020

Name ______________________________

Student ID #____________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:____________________________

1	16	
2	10	
3	12	
4	12	
Bonus	5	
Total	50	

- Your exam should consist of this cover sheet, followed by 4 problems and a bonus question. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 4 problems. Try not to spend more than 10 minutes on each problem.
- Show all your work and justify your answers.
- Your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use TI-30X IIS calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing calculators) are forbidden.
- The use of headphones or earbuds during the exam is not permitted.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (16 Points) True / False and Short Answer.

   Clearly indicate whether the statement is true or false and **justify your answer**.

   (a) **TRUE** / **FALSE** $\mathbb{R}^4$ is a subspace of $\mathbb{R}^6$.

   (b) **TRUE** / **FALSE** Let A be a 4×3 matrix. Then the nullity$(A) \geq 1$.

   (c) **TRUE** / **FALSE** Let A be a 4×3 matrix. Then $T(\vec{x}) = A\vec{x}$ could be one-to-one.

Give an example of each of the following. If there is no such example, write NOT POSSIBLE and **justify why it is not possible.** If you provide an example, you do not need to justify why the example works.

(d) **Give 3 examples** of matrices whose square is the identity, but is not itself the identity. (Find $A \neq I_2$ such that $A^2 = I_2$.)

(e) **Give an example** of a linear transformation $T : \mathbb{R}^5 \mapsto \mathbb{R}^5$ such that $\text{Range}(T) = \text{Ker}(T)$.

(f) **Give an example** of a 2×2 matrix that represents first scaling by 2, then rotating by $\frac{\pi}{4}$, then reflecting over the y-axis (in that order).

(g) **Give an example** of a 4×3 matrix A such that $T(\vec{x}) = A\vec{x}$ is onto.

2. (10 points)

   (a) Solve the following matrix equation for X. You may assume all matrices are square and invertible. You must show all of your work to receive credit.

$$AX = AXC - BC$$

   (b) Solve the following matrix equation for X. You may assume all matrices are square and invertible. You must show all of your work to receive credit.

$$X(AB - CD) = XA + BC$$

3. (12 points)

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by:

$$T(\vec{x}) = \begin{bmatrix} 2x_1 + x_2 + x_3 + x_4 \\ x_2 + x_3 + x_4 \\ 3x_3 + 3x_4 \\ 2x_4 \end{bmatrix} \tag{1}$$

(a) Show the the set of vectors in $\mathbb{R}^4$ that scale by 2 under the transformation T forms a subspace.

(b) Find a basis for the subspace from part (a). Show all of your work.

4. (12 Points) Let T be a linear transformation such that $T(\vec{x}) = A\vec{x}$ where A and its equivalent reduced row echelon form is given by

$$A = \begin{bmatrix} 1 & 1 & 4 & 2 \\ 1 & 4 & 1 & -1 \\ 0 & 3 & -3 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) What is the domain of T?

(b) What is the codomain of T?

(c) Give a basis for the column space of A that includes the vector $\begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}$. If this is not possible, explain why. If it is possible, show all of your work and explain your thinking.

(d) Is T onto? Justify your answer.

Bonus: (5 points) Find all possible matrices B such at $AB = I_2$ where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

(All such matrices would be called ”right inverses” of the matrix A. Working this problem shows that they are not unique when A is not square!)