MATH 308 N
Exam II
November 13, 2019

Name ____________________________________

Student ID #________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________

1	8	
2	14	
3	14	
4	14	
Bonus	5	
Total	50	

- Your exam should consist of this cover sheet, followed by 4 problems and a bonus question. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 4 problems. Try not to spend more than 12 minutes on each problem.
- Show all your work and justify your answers.
- Your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use TI-30X IIS calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing calculators) are forbidden.
- The use of headphones or earbuds during the exam is not permitted.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (8 Points) True / False and Short Answer.

   Clearly indicate whether the statement is true or false and **justify your answer** using only material we have covered in the course.

   (a) **TRUE / FALSE** If A and B are both invertible $n \times n$ matrices, then AB is invertible.

   (b) **TRUE / FALSE** $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

   Give an example of each of the following. If there is no such example, write NOT POSSIBLE and **justify your answer**.

   (c) **Give an example** of a matrix A such that $A^8 = I_2$, but $A^k \neq I_2$ for an integer k such that $0 < k < 8$.

   (d) **Give an example** of a linear transformation $T : \mathbb{R}^3 \mapsto \mathbb{R}^4$, where $T(\vec{x}) = A\vec{x}$ for some matrix A, such that $\text{Rank}(A) = 2$ and $\text{Nullity}(A) = 2$.

2. (14 Points) Let T be a linear transformation such that $T(\vec{x}) = A\vec{x}$ where A and its equivalent reduced row echelon form is given by

$$A = \begin{bmatrix} 1 & 3 & 21 & 22 \\ -1 & 1 & 3 & -2 \\ 1 & 1 & 9 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & 6 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) What is the codomain of T?

(b) Is the null space of A in the domain or codomain? Give a basis for Null(A).

(c) Is the column space of A in the domain or codomain? Give a basis for Col(A).

(d) Is the row space of A in the domain or codomain? Give a basis for Row(A).

(e) Is T one-to-one? Onto? **Justify your answer.**

3. (14 points)

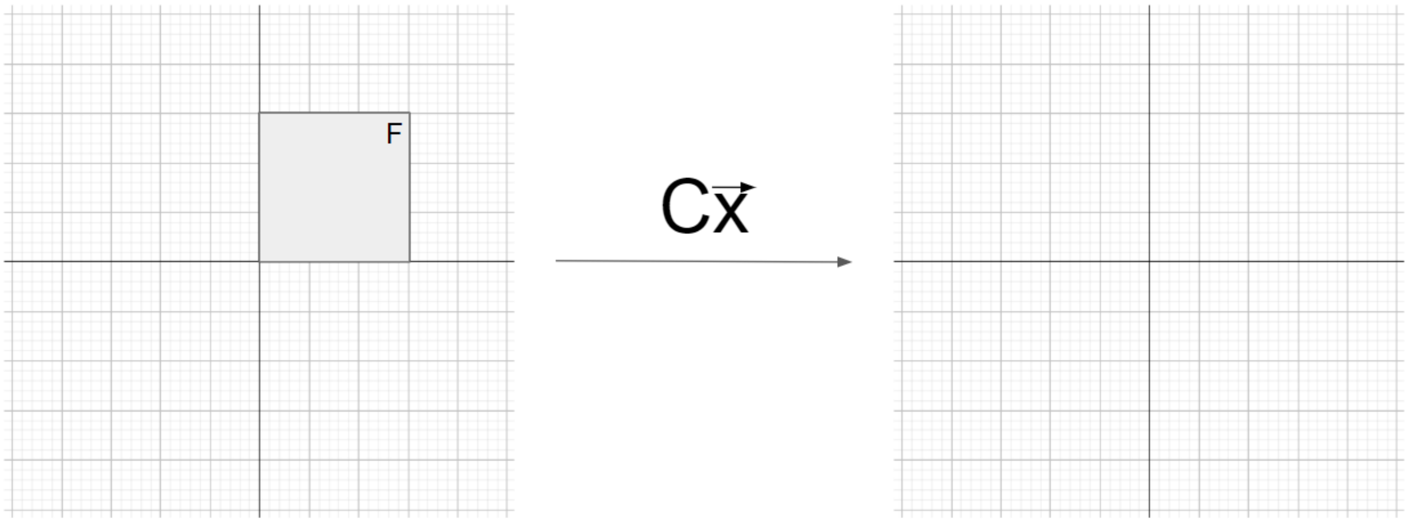
   (a) Produce a 2×2 matrix that reflects $\mathbb{R}^2$ over the y-axis.

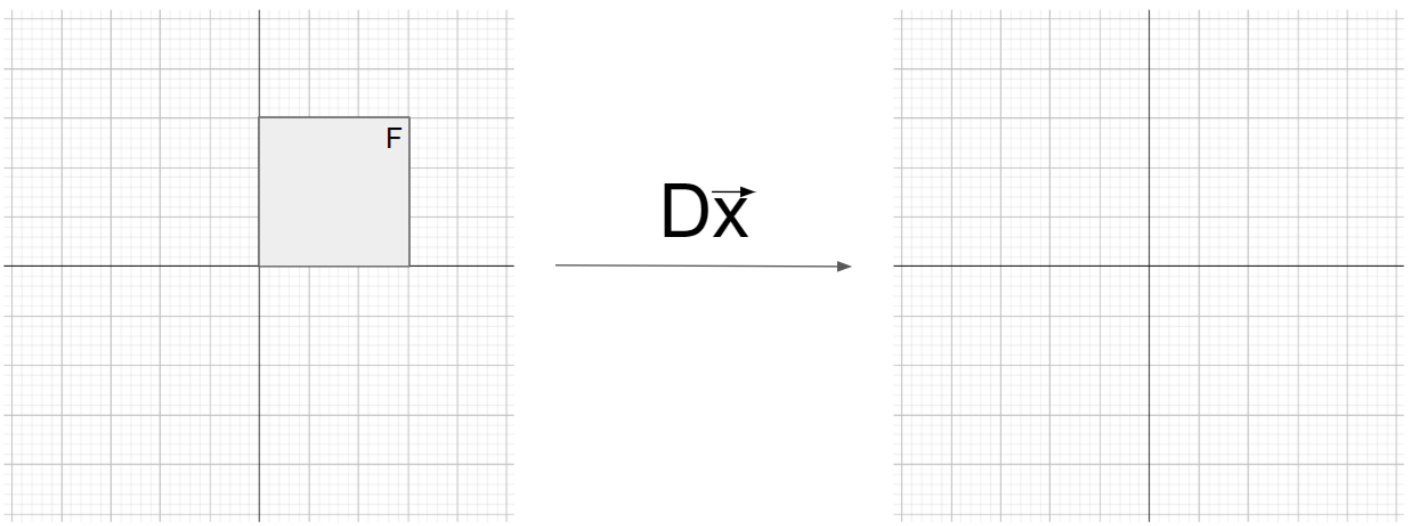
   (b) Produce a 2×2 matrix that rotates $\mathbb{R}^2$ by 270 degrees ($\frac{3\pi}{2}$ radians) counter-clockwise.

   (c) Compute the matrix that represents a reflection of $\mathbb{R}^2$ over the y-axis then a rotation by 270 degrees counter-clockwise, in that order. Call this matrix C.

   (d) Compute the matrix that represents a rotation of $\mathbb{R}^2$ by 270 degrees counter-clockwise, then a reflection over the y-axis, in that order. Call this matrix D.

(e) Complete the following drawings. Show where the unit square gets mapped and draw F with the correct orientation on the new square. Matrix C denotes the matrix from part c, and matrix D denotes the matrix from part d.





(f) **Fill in the blank.** This is a geometric proof of the fact that matrix multiplication is not ____.

4. (14 points)

(a) Let A be any square matrix A and λ any scalar. Show that the set S consisting of the vectors $\vec{v}$ such that $A\vec{v} = \lambda\vec{v}$ is a subspace.

(b) Now, let $\lambda = -2$ and let

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}.$$

Give a basis for S as given in part (a). You may use the back of this page if you need more space.

BONUS: (5 points) Short answer. To receive credit, you must fully justify your reasoning.

(a) Define a bracket operation on two matrices X and Y by $[X, Y] = XY - YX$. Now, let A and B be two matrices such that $(A+B)^2 = A^2 + 2AB + B^2$. Compute $[A, B]$.

(b) Now, let C and D be matrices such that $[C, D] = A$, where A is not the zero-matrix. What can we say about C and D?

(c) Summarize what this bracket operation is doing in terms of the algebraic structure we have developed for matrices.