MATH 308 M
Exam I
January 31, 2020

Name ______________________________________

Student ID #________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_________________________________

1	16	
2	10	
3	10	
4	14	
Bonus	5	
Total	50	

- Your exam should consist of this cover sheet, followed by 4 problems and a bonus question. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 4 problems. Try not to spend more than 10 minutes on each problem.
- Show all your work and justify your answers.
- Your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use TI-30X IIS calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing calculators) are forbidden.
- The use of headphones or earbuds during the exam is not permitted.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (16 Points) True / False and Short Answer (2 pages).

   Clearly indicate whether the statement is true or false. **If true, justify your answer. If false, provide a counterexample.**

   (a) **TRUE / FALSE** If $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is a linearly dependent set of vectors in $\mathbb{R}^5$, then $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\, \vec{u}_4\}$ is linearly dependent for any choice of $\vec{u}_4$ in $\mathbb{R}^5$.

   (b) **TRUE / FALSE** Let $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$ be a set of linearly dependent vectors in $\mathbb{R}^3$. Then $\vec{v_1} \in \text{span}\{\vec{v_2}, \vec{v_3}, \vec{v_4}\}$.

   Give an example of each of the following. If there is no such example, write NOT POSSIBLE. You **do not** need to justify that your example satisfies the desired conditions.

   (c) **Give an example** of a linear system of equations consisting of an infinite number of distinct equations and a finite number of variables with precisely one solution.

(d) **Give an example** of a system of equations with more variables than equations that has no solution.

(e) **Give an example** of a system of equations with no solution, but when one equation is removed, the new system has infinite solutions.

(f) **Give an example** of a matrix in echelon form with a pivot in every row where there are more columns than rows.

2. (10 Points)

(a) Clearly circle the sets of vectors below that permit a linear combination equal to $\begin{bmatrix} a \\ b \end{bmatrix}$ for any a, b real numbers. You are not required to show work on part (a). You may use your geometric intuition.

$$\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 10 \\ 0 \end{bmatrix} \right\},$$

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -6 \\ -10 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\},$$

(b) Assume you circled the correct sets in part (a). What is the span of each of the circled sets?

(c) Which of the circled sets are linearly independent sets? Which are not? **Justify your answer.**

(d) Which of the non-circled sets are linearly independent sets? Which are not? **Justify your answer.**

3. (10 points) Let $S = \{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$ be a set of vectors in $\mathbb{R}^4$ where

$$\vec{u}_1 = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} -5 \\ 4 \\ 2 \\ 0 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}, \quad \vec{u}_4 = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 2 \end{bmatrix}$$

(a) It turns out S is a linearly dependent set. Show this.

(b) Now write one of the vectors in the set S as a linear combination of the remaining vectors.

4. (14 points)

   (a) Find all values of a such that the set of vectors $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ are linearly dependent. Then find all values of a such that the set spans $\mathbb{R}^3$. If you use a shortcut, justify your work with a theorem.

$$\vec{v_1} = \begin{bmatrix} 2 \\ a \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ a \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}.$$

BONUS: (5 points) The span of the following vectors is a plane.

$$\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}2\\1\\-5\end{bmatrix}, \begin{bmatrix}3\\2\\-12\end{bmatrix}$$

Write the scalar equation of the plane.