

MATH 308 O
Exam I
April 22, 2020

Name _____

Student ID # _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	20	
2	10	
3	10	
4	10	
Bonus	5	
Total	50	

- Your exam should consist of this cover sheet, followed by 4 problems and a bonus question. Check that you have a complete exam.
- Pace yourself. You have 50 minutes to complete the exam and there are 4 problems. Try not to spend more than 10 minutes on each problem. You will have 10 minutes at the end of the exam to upload your solutions to Gradescope.
- Show all your work and justify your answers.
- Your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- This is an open book exam, however, you are not allowed to collaborate with anyone.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (18 Points) True / False and Short Answer (2 pages).

Clearly indicate whether the statement is true or false. **If true, justify your answer. If false, provide a counterexample or give justification.**

(a) **TRUE / FALSE** A set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent if and only if the first vector in the set, \vec{v}_1 , is in the span of the others vectors, $\{\vec{v}_2, \vec{v}_3\}$.

(b) **TRUE / FALSE** The zero vector, $\vec{0}$, can be a solution to a nonhomogeneous linear system of equations.

(c) **TRUE / FALSE** A system of equations with more variables than equations always has at least one solution.

Give an example of each of the following (and answer the question if needed). If there is no such example, write NOT POSSIBLE. You **do not** need to justify that your example satisfies the desired conditions, nor do you need to justify answers to the additional questions.

- (d) **Give an example** of a linear system of equations with exactly four variables with no solutions such that when you remove an equation (you can pick which one), the remaining system of equations has exactly one solution.

- (e) **Give an example** of a set of vectors that is linearly independent but does not span \mathbb{R}^2 . How many vectors can be in this set? Are there any other conditions?

- (f) **Give an example** of a set of vectors that spans \mathbb{R}^4 but is not linearly independent. What is the minimum number of vectors that must be in the set?

2. (10 Points) Identify which matrices are equivalent under row operations. Justify by listing the row operations (using notation from the lecture notes or clearly expressing each operation in words). Then, for matrices that are not equivalent, explain how you know.

$$\left\{ \begin{bmatrix} 3 & -6 & 9 \\ 0 & 2 & 8 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 & 2 \\ 6 & 6 & 4 \\ 1 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 4 \\ 0 & 3 & -10 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

3. (12 points) Recall that quadratic polynomials are polynomials of the form $y = a_0 + a_1x + a_2x^2$, where a_0 , a_1 and a_2 are any real numbers.
- (a) For what values of c are there infinitely many different quadratic polynomials such that $(2, 0)$, $(3, 0)$, $(c, 0)$ are points on the quadratic polynomial?
- (b) Using the same points as in part (a), for what values of c does there exist exactly one polynomial passing through the points? It turns out to be the same polynomial for all of these values. What is that polynomial?

4. (10 points) The following system of equations has one unique solution.

$$\begin{cases} x_1 + x_3 = 1 \\ x_2 - x_4 = -7 \\ x_1 - x_2 + x_4 = 15 \\ x_2 + x_3 + x_4 = -3 \end{cases}$$

- (a) Identify two methods you can use to show the system has one unique solution, neither of which involves actually solving the system. (Neither of your methods can be "solving the system using an augmented matrix.") Justify that your methods will work.
- (b) Use one of these two methods to show the system has one unique solution. (No points will be awarded for directly solving the system.)

BONUS: (5 points) Let A be a matrix whose columns are not linearly independent. If we solve for all solutions to $A\vec{x} = 0$, the solution will be of the form:

$$\vec{x} = s_1\vec{v}_1 + s_2\vec{v}_2 + \cdots + s_n\vec{v}_n$$

for some number of parameters s_1, s_2, \dots, s_n . Show that (with justification) the vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ are linearly independent.

Hint: Look at the solution to a concrete example, like the homogeneous system in Example 3 in lecture notes 2.3. Are the vectors in the solution linearly independent? Why? Can you generalize that argument to answer this question?