Lecture #6

3.1 Linear Transformations

DEF (linear transformation)

A function $T : \mathbb{R}^m \to \mathbb{R}^n$ is a $\boxed{\text{linear transformation}}$ if for all vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^m$ and all scalars r we have

(a) $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$

(b) $T(r\vec{u}) = rT(\vec{u})$.

EXERCISE Do you recall something that has property a? (see the THM on page 11 of Lecture #5.)

EXAMPLE 1 Let T be a linear transformation such that

$$T(\vec{u}_1) = \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} \quad \text{and} \quad T(\vec{u}_2) = \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix}.$$

For some $\vec{u}_1$ and $\vec{u}_2$ in $\mathbb{R}^5$. Find $T(4\vec{u}_1 - 3\vec{u}_2)$.

Just use properties of linear transformations above:

$$\begin{aligned} T(4\vec{u}_1 - 3\vec{u}_2) &= T(4\vec{u}_1 + (-3)\vec{u}_2) \\ &= T(4\vec{u}_1) + T(-3\vec{u}_2) \\ &= 4T(\vec{u}_1) + (-3)T(\vec{u}_2) \\ &= 4\begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} + (-3)\begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} 16 \\ -4 \\ -8 \end{bmatrix} + \begin{bmatrix} -3 \\ -3 \\ -18 \end{bmatrix} = \begin{bmatrix} 13 \\ -7 \\ -26 \end{bmatrix}. \end{aligned}$$

In the next example, we will give a concrete example of a linear transformation... a matrix with properties a) and b).

EXAMPLE 2

Let $A = \begin{bmatrix} 4 & -1 \\ -2 & 2 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

Let $r=2$, $s=-1$, and $T(\vec{x}) = A\vec{x}$. Verify that a) and b) holds for $r, s, \vec{u}$, and $\vec{v}$ by computing $T(r\vec{u}+s\vec{v})$ and $rT(\vec{u}) + sT(\vec{v})$.

First:

$$
\begin{aligned}
T(r\vec{u}+s\vec{v}) &= T\left(2\begin{bmatrix} -1 \\ -3 \end{bmatrix} + -1\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) \\
&= T\left(\begin{bmatrix} -2 \\ -6 \end{bmatrix} + \begin{bmatrix} -2 \\ -5 \end{bmatrix}\right) \\
&= T\left(\begin{bmatrix} -4 \\ -11 \end{bmatrix}\right) \\
&= A \cdot \begin{bmatrix} -4 \\ -11 \end{bmatrix} \\
&= \begin{bmatrix} 4 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} -4 \\ -11 \end{bmatrix} \\
&= \begin{bmatrix} -5 \\ -14 \end{bmatrix}
\end{aligned}
$$

Next:

$$
\begin{aligned}
rT(\vec{u}) + sT(\vec{v}) &= 2T\left(\begin{bmatrix} -1 \\ -3 \end{bmatrix}\right) + -1T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) \\
&= 2 \cdot A\begin{bmatrix} -1 \\ -3 \end{bmatrix} + -1 \cdot A\begin{bmatrix} 2 \\ 5 \end{bmatrix} \\
&= 2 \cdot \begin{bmatrix} 4 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ -3 \end{bmatrix} + (-1)\begin{bmatrix} 4 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix}
\end{aligned}
$$

$$= 2\begin{bmatrix}-1\\-4\end{bmatrix} + (-1)\begin{bmatrix}3\\6\end{bmatrix}$$

$$= \begin{bmatrix}-2\\-8\end{bmatrix} + \begin{bmatrix}-3\\-6\end{bmatrix}$$

$$= \begin{bmatrix}-5\\-14\end{bmatrix} \quad \checkmark$$

Note: the example does not show that the matrix gives us a linear transformation.

EXERCISE How could we show $T(\vec{x}) = A\vec{x}$ is a linear transformation for $A = \begin{bmatrix}4 & -1\\-2 & 2\end{bmatrix}$?

(Let $\vec{u}, \vec{v}, r, s$ be abstract vectors, and show $T(r\vec{u}+s\vec{v}) = rT(\vec{u}) + sT(\vec{v})$ for all vectors. i.e. Let $\vec{u} = \begin{bmatrix}u_1\\u_2\end{bmatrix}$ and $\vec{v} = \begin{bmatrix}v_1\\v_2\end{bmatrix}$ and compute.)

Show that T as above is a linear transformation.

It turns out that any transformation defined this way, i.e. $T(\vec{x}) = A\vec{x}$ for some matrix A, is always a linear transformation. Let's make a few definitions ... then we'll give this theorem.

DEF Suppose that A is a matrix with n-rows and m-columns. Then we say A is an $\boxed{n \times m \text{ matrix}}$ and that A has $\boxed{\text{dimensions}}$ $n \times m$. If $n = m$, then A is a $\boxed{\text{square matrix}}$.

<u>THM</u> Let A be an $n \times m$ matrix, define $T(\vec{x}) = A\vec{x}$. Then $T: \mathbb{R}^m \longrightarrow \mathbb{R}^n$ is a linear transformation.

<u>pf:</u> Let $\vec{u}, \vec{v}$ be arbitrary vectors in $\mathbb{R}^m$. Then

$$T(\vec{u}+\vec{v}) = A(\vec{u}+\vec{v}) \overset{(*)}{=} A\vec{u} + A\vec{v} = T(\vec{u}) + T(\vec{v}).$$

(<u>THM</u> on page 11 of Lecture #5)

Now, let r be a scalar, $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$, $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}_{n \times m}$.

Then

$$T(r\vec{u}) = A \cdot r\vec{u} = A \cdot \begin{bmatrix} ru_1 \\ ru_2 \\ \vdots \\ ru_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} ru_1 \\ ru_2 \\ \vdots \\ ru_m \end{bmatrix}$$

$$= \begin{bmatrix} ru_1 a_{11} + ru_2 a_{12} + \cdots + ru_m a_{1m} \\ ru_1 a_{21} + ru_2 a_{22} + \cdots + ru_m a_{2m} \\ \vdots \\ ru_1 a_{n1} + ru_2 a_{n2} + \cdots + ru_m a_{nm} \end{bmatrix}$$

$$= r \begin{bmatrix} u_1 a_{11} + u_2 a_{12} + \cdots + u_m a_{1m} \\ u_1 a_{21} + u_2 a_{22} + \cdots + u_m a_{2m} \\ \vdots \\ u_1 a_{n1} + u_2 a_{n2} + \cdots + u_m a_{nm} \end{bmatrix}$$

$$= r \cdot \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

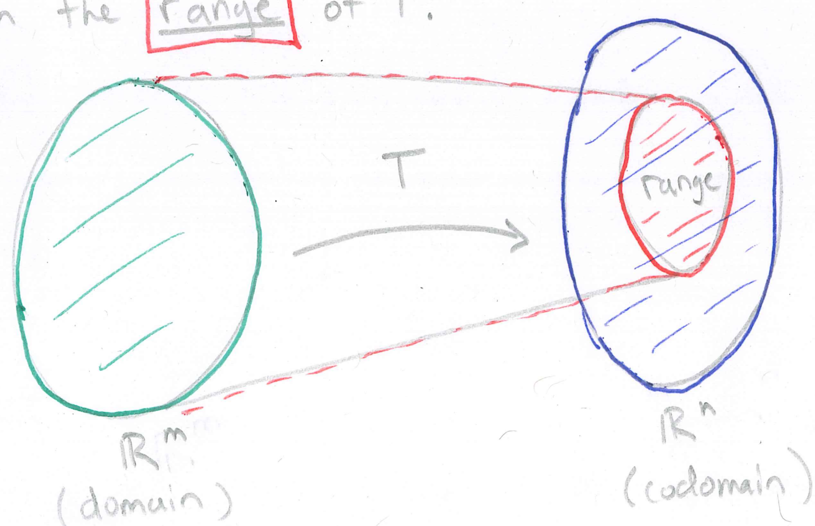
$$= rA(\vec{u}) = rT(\vec{u}).$$

✓

NOTE: You should understand this proof.

Next, we are going to focus a bit more on describing these maps, or transformations. What are some of the big picture properties worth considering?

DEF (Domain, Codomain, and Range)
Let T be a linear transformation, $T: \mathbb{R}^m \longrightarrow \mathbb{R}^n$.
Then the $\boxed{\text{domain}}$ is $\mathbb{R}^m$ (the vectors you give the transformation) and $\mathbb{R}^n$ is the $\boxed{\text{codomain}}$. If a vector $\vec{w}$ in the codomain is $T(\vec{x})$ for some $\vec{x}$ in the domain, we say $\vec{w}$ is in the $\boxed{\text{range}}$ of T.



"The range is where T maps stuff"

Note Some people call the range of T the image of T.

EXAMPLE 3

Let $A = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -5 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

Suppose $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ with $T(\vec{x}) = A\vec{x}$. Determine if $\vec{w}$ is in the range of T.

To determine whether or not $\vec{w}$ is in the range, we want to know if there is an $\vec{x}$ such that

$$A\vec{x} = \vec{w}.$$

i.e. ... we just use Gaussian Elimination.

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & 3 \\ 3 & 0 & -5 & 4 \end{array}\right] \underset{\text{(check!)}}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 4 & 3 \\ 0 & 6 & -17 & -5 \end{array}\right]$$

Back substitute! $x_3 = s$

$$6x_2 - 17s = -5$$

$$x_2 = \frac{-5}{6} - \frac{17}{6}s$$

$$x_1 - 2x_2 + 4x_3 = 3$$

$$x_1 - 2\left(\frac{-5}{6} - \frac{17}{6}s\right) + 4s = 3$$

$$x_1 = \frac{4}{3} + \frac{5}{3}s.$$

so,

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} \\ -5/6 \\ 0 \end{bmatrix} + s \begin{bmatrix} \frac{5}{3} \\ \frac{17}{6} \\ 1 \end{bmatrix}$$

and we see that $\vec{w}$ is the image of infinitely many $\vec{x}$ in $\mathbb{R}^3$!

The last example helped us think through what it means for a vector in the codomain to be in the range. We just looked at the equation $A\vec{x} = \vec{w}$. $\vec{w}$ is in the range as long as there is a solution to this system... i.e. $A\vec{x} = \vec{w}$ is a consistent linear system. We can also rewrite the equation..

$$\vec{w} = A\vec{x} = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 - 2x_2 + 4x_3 \\ 3x_1 + 0x_2 + -5x_3 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 \\ 3x_1 \end{bmatrix} + \begin{bmatrix} -2x_2 \\ 0x_2 \end{bmatrix} + \begin{bmatrix} 4x_3 \\ -5x_3 \end{bmatrix}$$

$$= x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -5 \end{bmatrix}.$$

i.e, we see that $\vec{w}$ must be in the span of the column vectors! We record this information in a theorem.

THM Let $A = [\vec{a}_1\ \vec{a}_2 \cdots \vec{a}_m]$ be an $n \times m$ matrix, and let $T: \mathbb{R}^m \longrightarrow \mathbb{R}^n$ be the linear transformation $T(\vec{x}) = A\vec{x}$. Then

a) The vector $\vec{w}$ is in the range of T if and only if $A\vec{x} = \vec{w}$ is a consistent linear system.

b) $\text{range}(T) = \text{span}\{\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_m\}$.

Now, a few more definitions:

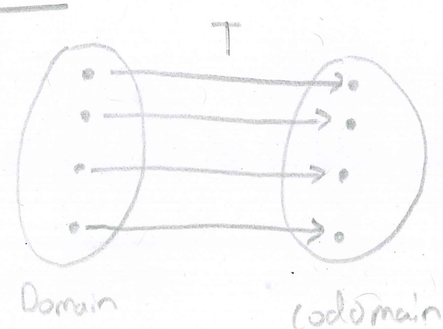
DEF (one-to-one, onto)

Let $T : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation. Then

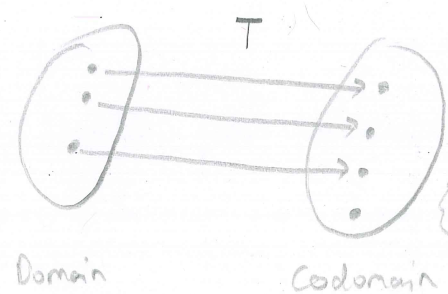
(a) T is **one-to-one** if for every vector $\vec{w}$ in $\mathbb{R}^n$, there exists at most one vector $\vec{u}$ in $\mathbb{R}^m$ such that $T(\vec{u}) = \vec{w}$.

(b) T is **onto** if for every vector $\vec{w}$ in $\mathbb{R}^n$, there is at least one vector $\vec{u}$ in $\mathbb{R}^m$ such that $T(\vec{u}) = \vec{w}$.

PICTURES

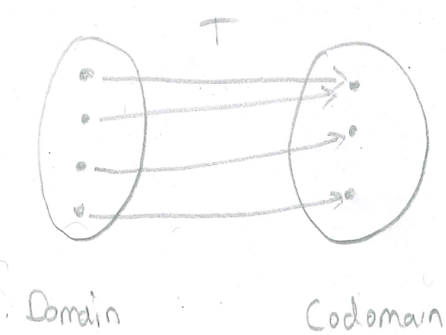


one-to-one AND onto



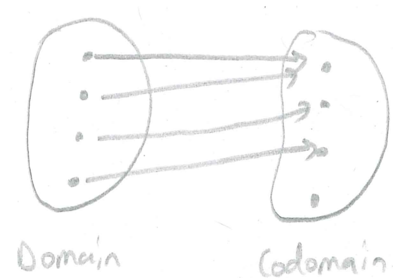
* there are points/vectors that don't get mapped to.

one-to-one, NOT onto



* every vector is mapped to, but some vectors get mapped to the same vector.

NOT one-to-one, but onto



NOT one-to-one, NOT onto

Notice, we could have defined one-to-one as follows.

DEF A linear transformation is $\boxed{\text{one-to-one}}$ if $T(\vec{u}) = T(\vec{v})$ implies $\vec{u} = \vec{v}$.

EXERCISE How are these two definitions equivalent?

THM Let T be a linear transformation. Then T is one-to-one if and only if the only solution to $T(\vec{x}) = \vec{0}$ is the trivial solution.

pf: First... notice that by definition, $T(\vec{0}) = \vec{0}$ can have at most one solution. By the properties of a linear transformation, we see it has exactly one:

$$\begin{aligned} \vec{0} = T(\vec{x}) - T(\vec{x}) &= T(\vec{x}) + {}^{-}T(\vec{x}) \\ &= T(\vec{x}) + T(-\vec{x}) \\ &= T(\vec{x} - \vec{x}) \\ &= T(\vec{0}). \end{aligned}$$

You should know this about linear transformations!

(This works by taking any $\vec{x}$ in the domain!)

This shows that being one-to-one implies $T(\vec{0}) = \vec{0}$.

Then, notice

Fix

Conversely, suppose $T(\vec{x}) = \vec{0}$ has only the trivial solution, $\vec{x} = \vec{0}$. If $T(\vec{u}) = T(\vec{v})$, then

$$0 = T(\vec{u}) - T(\vec{v}) = T(\vec{u} - \vec{v}) = T(\vec{0})$$

so we see $\vec{u} = \vec{v}$.

$\square$

EXAMPLE 4

Let $A = \begin{bmatrix} 4 & -1 \\ -2 & 2 \end{bmatrix}$. Determine if $T(\vec{x}) = Ax$ is one-to-one.

$\Rightarrow$ use the last Theorem! Check to see if there is only one solution to $A\vec{x} = \vec{0}$:

$$\left[\begin{array}{cc|c} 4 & -1 & 0 \\ -2 & 2 & 0 \end{array}\right] \underset{\text{(check!)}}{\sim} \left[\begin{array}{cc|c} -2 & 2 & 0 \\ 0 & 3 & 0 \end{array}\right]$$

fix

$\Rightarrow 3x_2 = 0 \Rightarrow x_2 = 0$

$-2x_1 + 2x_2 = 0 \Rightarrow x_1 = 0$.

So we see only one solution, $\vec{x} = \vec{0}$. Thus, T is one-to-one.

Notice, we are using the same machinery that we used before when thinking about span and linear dependence. We can connect these concepts:

THM Let A be an $n \times m$ matrix and define $T: \mathbb{R}^m \to \mathbb{R}^n$ by $T(\vec{x}) = A\vec{x}$. Then

(a) T is one-to-one if and only if the columns of A are linearly independent.

(b) If $A \sim B$ and B is in echelon form, then T is one-to-one if and only if B has a pivot position in every column.

(c) If $n < m$, then T is not one-to-one.

Similarly, we have the analogous theorem for "onto".

<u>THM</u> Let A be an $n \times m$ matrix and define $T: \mathbb{R}^m \to \mathbb{R}^n$ by $T(\vec{x}) = A\vec{x}$. Then

(a) T is onto if and only if the columns of A span the codomain.

(b) If $A \sim B$ and B is in echelon form, then T is onto if and only if B has a pivot position in every row.

(c) If $n > m$, then T is <u>not</u> onto.

Notice, both of the previous theorems extend theorems from 2.2 and 2.3. In 2.2, we discovered that if a set of vectors span a space, the corresponding matrix equation (where the vectors are columns in the matrix) has <u>at least</u> one solution ($A\vec{x} = \vec{b}$ always has <u>at least</u> one solution for any $\vec{b}$). This means A, as a linear transformation, is <u>onto</u>. Similarly, if the column vectors are linearly independent, there is <u>at most</u> one solution to $A\vec{x} = \vec{b}$, so A, as a linear transformation, is one-to-one.

Now, notice that we have showed that if you have an $n \times m$ matrix A, then $T(\vec{x}) = A\vec{x}$ is a linear transformation. It turns out that all linear transformations can be represented as a matrix.

THM Let $T: \mathbb{R}^m \to \mathbb{R}^n$. Then T is a linear transformation if and only if $T(\vec{x}) = A\vec{x}$ for some $n \times m$ matrix A.

pf: The theorem on page 4 of these notes proves one direction. For the other, assume T is a linear transformation. Let

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \ldots, \quad e_m = \left.\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}\right\} m \text{ entries}$$

be vectors in $\mathbb{R}^m$. Let $A = [T(\vec{e}_1) \cdots T(\vec{e}_2)]$.

($T(\vec{e}_1)$ is the linear transformation applied to $\vec{e}_1$, so A is made up of column vectors $T(\vec{e}_1), T(\vec{e}_2), \ldots, T(\vec{e}_m)$.)

Now, notice that any $x \in \mathbb{R}^m$ can be written as:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \cdots + x_m \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$= x_1 \vec{e}_1 + x_2 \vec{e}_2 + \cdots + x_m \vec{e}_m.$$

Then, using the properties of linear transformations,

make sure you understand each step here!

$$
\begin{aligned}
T(\vec{x}) &= T\left(x_1\vec{e}_1 + x_2\vec{e}_2 + \cdots + x_m\vec{e}_m\right) \\
&= x_1 T(\vec{e}_1) + x_2 T(\vec{e}_2) + \cdots + x_m T(\vec{e}_m) \\
&= A\vec{x}.
\end{aligned}
$$

So, we see that we can represent T by a matrix.

$\boxtimes$

<u>NOTE</u> This theorem does not say A is unique ... and in fact, before we got A, we had to choose a very special set of vectors, $\vec{e}_1, .., \vec{e}_m$, in $\mathbb{R}^m$.

<u>EXERCISE</u> What was the key property of the set of vectors $\vec{e}_1, .., \vec{e}_m$ that we used? Can you think of any other vectors with this property?

<u>EXERCISE</u> Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ be defined by

$$
T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + x_3 \\ -x_1 + 2x_2 \\ x_1 - 3x_2 + 5x_3 \\ 4x_2 \end{bmatrix}.
$$

Show that T is a linear transformation by finding the matrix A such that $T(\vec{x}) = A\vec{x}$.

Now, we return to "one-to-one" and "onto". The theorems we gave earlier connected these new notions to span and linear independence. We can now consider a special case, where A is a square matrix, and upgrade our unifying theorem.

THM (Unifying Theorem, version 2)

Let $S = \{\vec{a}_1, .., \vec{a}_n\}$ be a set of n vectors in $\mathbb{R}^n$, let $A = [\vec{a}_1 \cdots \vec{a}_n]$, and let $T: \mathbb{R}^n \to \mathbb{R}^n$ be given by $T(\vec{x}) = A\vec{x}$. Then the following are equivalent:

a) S spans $\mathbb{R}^n$.

b) S is linearly independent.

c) $A\vec{x} = \vec{b}$ has a unique solution for all $\vec{b}$ in $\mathbb{R}^n$.

(*) d) T is onto

(*) e) T is one-to-one.

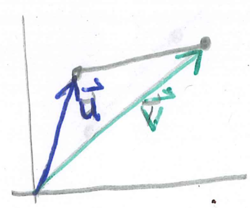
(In this setting, T is onto if and only if T is one-to-one!)

A little geometry

so far, we have focused on some of the more abstract properties of linear transformations. However, when thought of as a matrix, the linear transformation can be interpreted geometrically. Below, we'll highlight a few important matrices for which you should understand how they act "geometry." First though, lets highlight one of the key geometric properties:

PROPERTY A linear transformation takes lines to lines (i.e., it maps lines to lines)

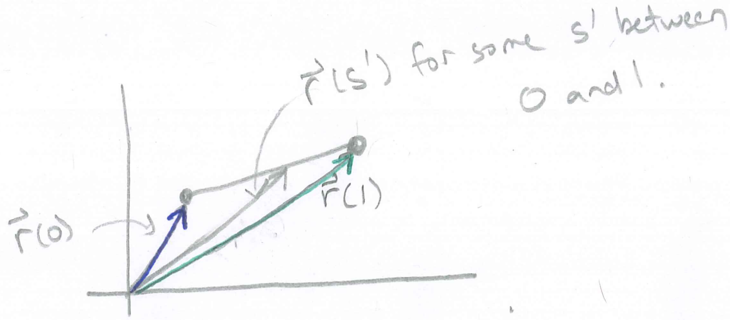
For a proof of this, we'll parametrize a line segment.



Notice, there is a vector pointing to each end of the line segment (true in more than just 2-D!), $\underline{\underline{u}}$ and $\underline{\underline{v}}$. We can parametrize the points along this line:

$$\vec{r}(s) = (1-s)\vec{u} + s\vec{v} \quad , \quad 0 \le s \le 1$$

then, as s increases, $\vec{r}(s)$ points to points on the line:



Now, let's transform all the vectors along the line:

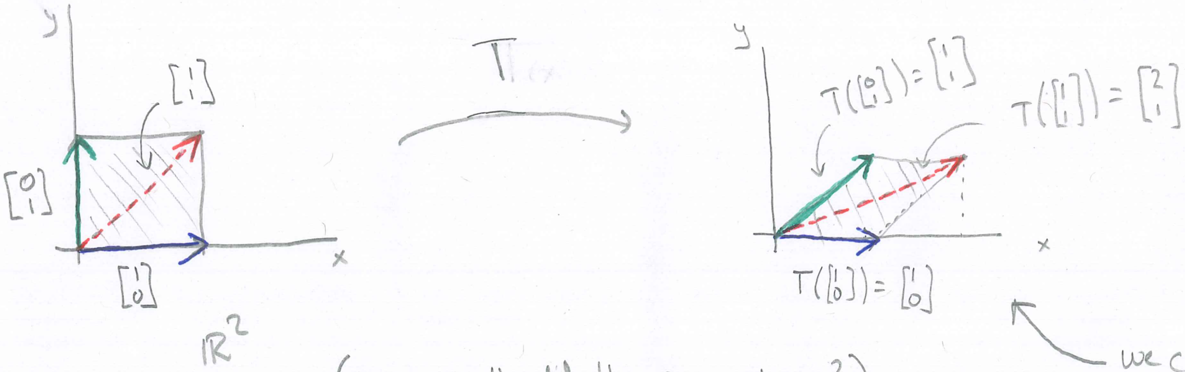
$$T\left((1-s)\vec{u} + s\vec{v}\right) = T\left((1-s)\vec{u}\right) + T(s\vec{v})$$
$$= (1-s)\,T(\vec{u}) + s\,T(\vec{v}).$$

Since $T(\vec{u})$ and $T(\vec{v})$ are just vectors in the codomain, $(1-s)\,T(\vec{u}) + s\,T(\vec{v})$ is a parametrization of a line segment between $T(\vec{u})$ and $T(\vec{v})$! i.e. any line segment must go to a line segment in the codomain.

Next, we can use this property to understand how matrices are mapping vectors.

Notice, if we map the unit square with a matrix, we get a really good idea of what the matrix is doing.

For example, let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, and notice $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ x \end{bmatrix}$, so for $T(\vec{x}) := A\vec{x}$, $T: \mathbb{R}^2 \to \mathbb{R}^2$, we see the unit square map as follows:



we call this a "shear"

(QUESTION: How did the area change?)

EXERCISE Do the same for the following matrices.

(1) $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$ (3) $\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

(2) $\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$

- How would you describe each one geometrically? As a shear? Reflection? Rotation?

(5) $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$

- What's weird about this one?

Here are a few "forms" a matrix can have which results in recognizable geometric transformations:

(it's one of these...)

$\boxed{\text{Rotation (by } \theta)}$: $\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$

$\boxed{\text{Reflection (over x-axis)}}$: [EXERCISE]

$\boxed{\text{Shear}}$: $\begin{bmatrix}1 & a\\0 & 1\end{bmatrix}$

$\boxed{\text{Reflection (over y-axis)}}$: [Exercise!]