Week 1 Logical Reasoning Review

1. Assume the following statement is true: Monkeys are green whenever the moon is purple. Which of the following statements is also true? Which are false?

   - If the moon is purple, monkeys are green.
   - If monkeys are green, the moon is purple.
   - It is necessary for the moon to be purple in order for monkeys to be green.
   - If you see a green monkey, the moon is purple.
   - The moon is not purple implies the monkeys are not green.
   - The monkeys are not green implies the moon is not purple.

2. Assume the following statement is true: It is sufficient to know that the window framing is rusting in order to know the window framing paint is cracked. Which of the following statements are true? Which are false?

   - If the paint is cracked, the window framing is rusting.
   - The windows rust whenever the paint is cracked.
   - If the paint is not cracked, the window framing is not rusted.
   - The window framing paint is cracked whenever the window framing is rusting.
   - The window framing paint is not cracked whenever the window framing is not rusting.
   - If the window frames are not rusting, the window frame paint is not cracked.

3. Negate the following sentence: Getika is 24 years old and Anush does not enjoy playing board games. (Hint: think about the truth table for ”and” and ”or”.)

4. Negate the following sentence: Sarah is an engineer or Bob is a teacher. (Hint: think about the truth table for ”or” and ”and”.)

5. Negate the following sentence: I have seen a cat without a smile, but never a smile without a cat! (Hint: Think of ”but” as ”and”.)

6. What are the contrapositives of the following conditional statements:

   - If n is a positive integer, then $n + 1$ is a positive integer.
   - If $n > 2$, then $n^2 - 1 > 0$.
   - $n = 2$ is necessary for $n^2 - n - 2 = 0$.
   - $n = 2$ is sufficient for $n^2 - n - 2 = 0$.

7. What are the converses of the following conditional statements? Are they true?

   - If n is a positive integer, then $n + 1$ is a positive integer.
   - If $n > 2$, then $n^2 - 1 > 0$.
   - $n = 2$ is necessary for $n^2 - n - 2 = 0$.
   - $n = 2$ is sufficient for $n^2 - n - 2 = 0$.

8. A visitor to a city whose inhabitants always tell the truth or always lie encounters an islander who makes the following two statements:

   - I love Bertha.
   - If I love Bertha, then I love Mathias.

   What can you deduce? (Careful: what does the truth table for a conditional statement look like?)

SPOILER ALERT: Answers on the next page.

Answers:

1.
   - T
   - F
   - F
   - F
   - F
   - T
2.
   - F
   - F
   - T
   - T
   - F
   - F
3. Getika is not 24 years old or Anush enjoys playing board games.
4. Sarah is not an engineer and Bob is not a teacher.
5. I have not seen a cat without a smile or I have seen a smile without a cat.
6.
   - If $n+1$ is not a positive integer, then n is not a positive integer.
   - If $n^2 - 1 \leq 0$, then $n \leq 2$.
   - If $n \neq 2$, then $n^2 - n - 2 \neq 0$.
   - If $n^2 - n - 2 \neq 0$, then $n \neq 2$.
7. Note: for the Canvas quiz, you do not need to be able to identify the truth or falseness of the statements, but you do need to be able to identify the converse.
   - If $n+1$ is a positive integer, then n is a positive integer. (False! Even if you call 0 positive, -1 is not.)
   - If $n^2 - 1 > 0$, then $n > 2$. (False! n could be less than -1, or between 1 and 2.)
   - If $n = 2$, then $n^2 - n - 2 = 0$. (True. Plug in $n = 2$.)
   - If $n^2 - n - 2 = 0$, then $n = 2$. (False! n could be -1.)
8. So, it turns out that this city dweller must be telling the truth. Assume for a moment that the first statement is true. Then the second statement *could* be true. Maybe it turns out that the city dweller loves both Bertha and Mathias. Now, assume that the first statement is false. This means that the city dweller is lying, so both statements must be false. Let's check whether or not the second statement could be false. Since "I love Bertha" is false, that means we have a "vacuous condition", i.e. we are looking at an empty set. Since the person loves Bertha, the world in which the person does not love Bertha does not exist! So, the conditional statement is true, regardless of whether the necessary condition is true or false. (Check out the explanation at the end of the lecture notes regarding the "truth table" for a conditional statement.) Just FYI, this particular property will not be tested on the Canvas quiz, but you should be aware of it.